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Collusion in Hierarchical Agency

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Abstract: We study a model where shareholders can use auditors' reports to contract with a privately informed manager but the manager can bribe the auditors to manipulate their reports. Such auditors are useful if they have good information and the liability of the manager is high.

In the optimal contract under collusion, even with unbounded punishments and costless auditing, production does not reach its optimal level. Raising the punishment for the manager raises the bribe he is willing to offer the auditor raising the cost of preventing collusion. When liability grows without bound and part of the punishment is non-transferable maximum deterrence will not be optimal .

To model cross-checking mechanisms, we distinguish internal (costless but may collude with the manager) from external (costly but never collude) auditors. We prove that the optimal contract might specify random external audits. We consider a self-interested external auditor and find that the optimal contract is unchanged.

Finally, we present a model where allowing collusion is the optimal strategy for the principal.

Collusion in Hierarchical Agency

Introduction

In recent years agency theory has begun to study the design of multi-person organizations by a principal who is interested in achieving performance from self-interested, strategic agents who posess private information. For instance, the board of directors of a firm must design incentive schemes for the managers and other employees. These employess know things about themselves, their environment or their actions which are unobservable to third parties.

The research in this area has by and large ignored the possibility of collusion of various sorts of people within the agency. Economic models in general, rarely consider coalitions with side-contracting power in the design of incentive schemes. Problems of side-payments, however, have been extensively recorded by sociologists and other behavioral scientists; furthermore, even if no evidence of collusion had been found its mere possibility would still constrain the set of feasible contracts.

A major exception to this kind of research is Tirole [1986], who has identified and studied efficiency losses that can result from collusion with side-payments in a principal-supervisor-agent hierarchy. He notes that coalitions with (implicit) side-contracts are more likely to form in long-term relationships through a reputation-like mechanism. These relationships are a mitigated blessing: repetition enhances productivity as well as opportunities for collusion. Although job-rotation and other impediments of prolonged interactions may be effective for disrupting coalitions, they may also be dear in terms of efficiency losses.

In this paper, we consider a different way of preventing collusion; creating an alternative source of information. Our suggestion is to introduce a second supervisor (in a horizontal relationship with the first) whose purpose is to discourage deviant coalitions. This additional supervisor is involved in a short-term contract with the hierarchy and thus less prone to organizational-culture pressure; he is also less efficient because he lacks specific experience and knowledge of the job.

An illustration of this phenomenon is the coexistence of internal and external auditing. Internal auditors provide management and shareholders with valuable information concerning not only the financial situation of the firm but also other aspects outside the accounting area. Indeed, internal auditors are usually involved in evaluating the efficiency of the firm's operating division. Labels such as "operational audits," "management audits," or "performance audits" are used to describe this kind of audit[1].

Because they "live in the firm," internal auditors possess high quality information and, since they also provide non-auditing services (e.g. tax assistance), the opportunity cost of their auditing function is low.

External auditors, in contrast, have poorer information about the firm and are more costly. Although their use can still be justified on "informational grounds," our intuition is that the additional information they provide does not account for the extent of their employment in the real world. In addition, most external operational audits are conducted on a random

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1 While financial audits are limited to evaluating the fairness of financial statements, operational audits involve any activities for which operational effectiveness can be evaluated (see, for instance, Arens and Loebbecke [1988]). Although the accounting literature links financial audits to adverse selection and operational audits to moral hazard, we study a model with both hidden information and hidden action which is more easily exposited in terms of an operational audit. However, it is possible to interpret our model in terms of a financial audit as well.

basis[2]. If one wants to argue that external auditors are hired only for their information, this randomness seems hard to rationalize.

In this paper we present a model in which we show that external auditing can be better explained by its role in enhancing the independence of internal auditors (See Antle [1984]). Internal auditors are usually hired (and fired) by the management, hence they are more likely to suffer pressures to give more favorable performance treatment or, in the case of a financial audit, to "window-dress" the financial statements in favor of the managers. This problem is of significant concern to regulatory agencies and the accounting profession[3]. External auditors are usually hired by the shareholders (or the audit committee of the Board of Directors), and are therefore more independent from the manager[4].

The optimal auditing policy involves a trade-off between internal auditors with good information at low cost but subject to management pressures, and external auditors with poor information at high cost but with unbiased reports.

In our model, the internal auditor and the manager can falsify evidence about the manager's performance. First we prove that, when his information is very reliable, the internal auditor is still useful for the principal. Then we introduce an external auditor who reports

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2 Regarding external financial audits, most firms are legally required by the SEC to perform them annually. One implication of this paper is that this annual requirement might not be optimal.

3 See the various reports of the Moss Committee (U.S. Congress [1976]), the Metcalf Committee (U.S. Congress [1977a], [1977b]), the Dingell Committee (U.S. Congress [1985]), the Securities and Exchange Commission accounting series releases (SEC ASR 165 [1974], ASR 247 [1978], ASR 250 [1978], ...) and the Cohen Comission's Report on Auditors' Responsibilities [1978]. An extensive discussion of this problem can be found in Mangold Nancy R. [1988]

4 A reputation of independence being an essential asset for an external audit firm, some of these firms have also created organizations (with voluntary membership) to improve the quality of auditing practice. For instance, the "SEC Practice Section" of the AICPA imposes stringent auditing standards on its members (partner rotation, concurrent partner review, proscription of certain services, mandatory peer review, etc.)

truthfully. For simplicity, we assume that he has the same information as the internal auditor but is more costly[5]. Therefore without the possibility of collusion between the manager and the internal auditor, the external auditor's services have no value. Depending on the values of the parameters of the problem, we find that the external auditor may or may not be hired in the optimal auditing scheme with collusion. If external audits are implemented at all, they are performed on a random basis. Last, we present a model where it may be optimal for the principal to allow collusion.

Another contribution of this paper is to show that, under collusion, even when unbounded punishments for the agent are available, the optimal contract will not specify efficient production (inefficiencies in production are maintained at every level of punishment). In addition to their deterrent effect, higher punishments induce higher bribes which are more difficult to prevent. This creates a situation analogous to one where using the internal auditor costs the principal a fixed proportion of the punishment he imposes on the agent.

Literature Review

Principal-agent theory has paid considerable attention to the incentive problems arising in two-layer hierarchies, i.e., one principal and one agent (Baron-Myerson [1982], Maskin-Riley [1984], Laffont-Tirole [1986]). With few exceptions, the standard model has been extended naively to incorporate a third layer, usually a supervisor, as a "third arm" for the principal (Baron-Besanko [1984]). This description of the supervisor ignores the possible conflict of interest between him and the principal. If their objectives differ, a new incentive problem arises and the self-interested supervisor must be adequately motivated to act on behalf of the principal. Demski and Sappington [1987] studied the case of a supervisor

5 Assuming that the external auditor has worse information than the internal would be more realistic but it would complicate the presentation of the results without adding to their understanding.

subject to moral hazard in a consumer-regulator-firm hierarchy. But, in their model, they do not consider the possibility of coalition-formation. Baiman, Evans and Noel [1987] studied the gain of transferring the adverse selection problem from the agent to the auditor but, again, no collusion is allowed in their model.

The principal can also increase his utility by using alternative sources of information. Hart [1983] formalized this idea in a model where product market competition can alleviate the incentive problem: the agent works harder when competition increases[6]. However, Scharfstein [1988] showed that this result is highly dependent on the agent's preferences; a slight alteration of the agent's utility function can make competition harmful.

The economic intuition of these models is similar to the one of the tournament literature. A tournament is a situation in which an agent's payment depends only on his output or rank relative to other competitors (Green-Stockey [1983], Lazear-Rosen [1981], Nalebuff-Stiglitz [1983]). But all of the schemes discussed in these papers are not incentive compatible if agents can collude.

The literature on implementation has studied the problem of preventing unwanted (collusive) equilibria. When the direct revelation game has multiple equilibria one needs to resort to non-revelation or indirect mechanisms. This has been done in three ways: (1) Take the revelation mechanism and construct an indirect mechanism in which the unwanted equilibria are knocked out by giving agents additional ("nuisance") strategies (Maskin [1977], Ma, Moore and Turnbull [1988]). (2) Change the payoffs to one of the agents so that his desirable equilibrium strategy becomes a dominant strategy (Demski and Sappington [1984] and Demski, Sappington and Spiller [1988].) (3) Add stages to the

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6 Contestability theory (Baumol, Panzar and Willig [1982]) claims that the threat of competition by itself can make a natural monopoly efficient. On the effects of potential competition see also Gilbert [1989].

mechanism, and impose the requirement that off-the-equilibrium-path strategies (threats) must be credible (Moore and Repullo [1988]).

Ma, Moore and Turnbull [1988] study the case in which a principal hires two agents to work in a correlated environment. They point out that the standard technique of making each agent's reward dependent on the other's performance has unsatisfactory features. Namely, the agents strictly prefer to play equilibrium strategies other than those desired by the principal. To knock-out these undesired but incentive-compatible equilibria their (indirect) mechanism uses one agent to police the other. Their technique differs from the one of Demski and Sappington [1984] in that while Demski and Sappington give the right incentives to the "police" agent by making the principal's desired choice a dominant strategy, Ma, Moore and Turnbull offer him a set of additional output levels from which to choose. While Demski and Sappington strengthen the principal's constraint set -and thus reduce his profits- Ma, Moore and Turnbull use the standard constraints and the costless technique of adding strategies which will never be played in equilibrium.

Moore and Repullo [1988] consider situations where agents are completely informed of the state of the world while the designer is completely ignorant of it. In these cases, the Revelation Principle has almost no cutting power and they set up a stage mechanism which has a unique (the desired one) sub-game perfect equilibrium.

The issue they do not consider is that although their Bayes-Nash equilibrium is unique and coalition-proof in the sense of Bernheim, Peleg and Whinston [1987] it need not be stable against coalitions when side-payments are allowed. In other words, if side contracts are feasible the agents might deviate together even though the outcome they play was not an equilibrium of the game set up by the principal's contract.

Grossman-Hart [1986] and Williamson [1985] discuss the difference between internal and external auditing in their studies of integration and the limits of firms. Grossman-Hart

argue that "integration in itself does not make any new variable observable to both parties. Any audits which an employer can have done of her subsidiary are also feasible when the subsidiary is a separate company." Williamson replies that interfirm auditing cannot be presumed to be as effective as intrafirm auditing: "[i]f a stronger mutual interest in organizational integrity can be presumed among members of an integrated organization than would exist between independent trading units [...] then internal auditors can expect to receive greater cooperation [...] than can be presumed when auditing across an autonomous ownership boundary is attempted." Williamson, however, recognizes that internal auditing is subject to corruption and points out how important is the question we address in this paper: "are the flaws of internal auditing remediable?"

Baiman, Evans and Nagarajan [1989] also study a model of auditing with collusion. Their approach is quite different from ours. The manager does not take any effort; he transfers the realization of a random output to the principal. The principal can use a costly auditor who observes output perfectly. The manager and the auditor can collude but the authors do not model the incentives for collusion; a move by "nature" determines whether self-enforcing collusive arrangements are feasible. Finally, they do not distinguish between internal and external auditors.

The Model

We study a three-layer hierarchy consisting of a principal, an auditor and a manager. The principal owns the vertical structure; the manager runs a productive unit with private information about its efficiency; the auditor collects information for the principal. Following Tirole [1986] we assume that the principal lacks either the time or the knowledge required to supervise the manager and that the auditor lacks either the time or the resources required to run the vertical structure.

Tirole relies on a third axiom which rules out the possibility of dividing the auditing job among several auditors. This allows the auditor and the manager to collude without any risk of being discovered. The point of this paper is to explore the consequences of introducing a second auditor together with a system of rewards and punishments to prevent deviant coalitions.

The first auditor —*internal*— performs several other tasks inside the firm and has to be hired anyway; we assume that the opportunity cost of him auditing is zero. By conducting an audit, the internal obtains information about the privately known productivity of the manager. "Living in the firm", the internal is likely to collude with the manager. The second auditor —*external*— serves only to perform his audit; we assume that he is "collusion-free" (he always reports truthfully), that his services are costly and that his information is perfectly correlated with the internal's information.

Players

Four people interact in this agency relationship: the principal, the internal auditor, the external auditor and the manager. All are assumed to be risk neutral.

The manager exerts effort e which, together with a productivity parameter θ, determines output x

$$x = \theta + e$$

We assume that the cost for the manager of exerting effor g(e) is an increasing, convex function. For simplicity we take

$$g(e) = e^2/2$$ [7]

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7 We have obtained the same qualitative conclusions in an earlier version of this paper (available upon request) considering a more general specification of g(e).

The output belongs to the principal who compensates the manager with a transfer t. The manager maximizes his payoff

Manager's payoff = t - g(e)

Since the manager signs the contract after learning the realization of θ, he must receive his expected reservation utility in any state of the world. (We assume that is optimal for the principal to employ the manager even when the lowest θ occurs.)

The internal auditor observes a signal s imperfectly correlated with θ. He receives a wage w from the principal, and his objective is to maximize his expected monetary income. The external receives the *same* signal s about θ and reports it truthfully to the principal. His services are provided at a cost z.

The principal receives the residual profits of the vertical structure and pays (or punishes when it is called for) the other members of the hierarchy. His objective is to maximize expected profits

$$E(\pi) = E(x - t - w - z + \text{punishments})$$

Uncertainty and information structure

We assume that θ can take only two values, $\theta_1 < \theta_2$. θ_1 obtains with probability q and θ_2 with probability (1 - q). While θ and e are private information of the manager, we will suppose that the output x (= θ + e) is publicly observable (and verifiable).

The internal and the external obtain an identical signal s imperfectly correlated with θ The signal can be s_1 or s_2 which occur with the conditional probabilities described in Table 1:

	s_1	s_2
θ_1	r	1 - r
θ_2	1 - r	r

Table 1: Correlation of true productivity and auditors' signal

We also assume that this signal is observable for the manager. The auditee usually knows what records were examined by the auditor and can deduce the inferences they support.

The process we have in mind when we say that the two auditors receive a signal imperfectly correlated with the true state of productivity is the following. When conducting an audit, the auditor does not have the material possibilities to examine all the firm's records, so he selects and examines only a sample of them and makes inferences regarding the situation of the firm which he then reports to the principal.

Collusion and Audit Technology

If the internal and the manager collude, they can forge evidence. For instance, suppose the internal tries to determine whether most of the firm's customers have bad credits or not. If the manager and the internal collude, the manager can indicate to the internal the bad debtors and, even though most of the customers are diligent payers, the coalition can present verifiable evidence of low productivity due to high financial charges. But if an external conducts an audit, he will obtain verifiable evidence of high productivity and the principal will know that the internal auditor and the manager colluded [8]. The principal can then punish both the internal auditor and the manager.

8 If the internal and external received imperfectly correlated signals, the inference would be subject to mistakes.

Note that we rule out the possibility of blackmail by the internal. If he gets a signal favorable to the manager, he can not threaten the manager with revelation of a less favorable signal since he needs his help to falsify evidence. In that sense, information is neither completely "soft" nor "hard". By this we mean that the internal's report can not convey information to the principal in a credible way when it announces a bad state of productivity but is verifiable when it announces high productivity.

In Tirole's model the supervisor can only hide information (reporting he observed "nothing") but not forge evidence. An auditor's report is usually a point estimate of the cost of the firm or a acceptance/rejection of the firm's financial report, therefore "nothing" is not a realistic alternative for his report. (See Shibano [1988]). It may be that some observations lead to a posterior distribution that is identical to the prior. These uninformative observations are formally equivalent to observing "nothing," so that our information structure is not inconsistent with Tirole's.

We will only study collusion between the manager and the auditor since they constitute the single nexus with ability to manipulate the information received by the rest of the vertical structure.

To avoid the distracting problem of bargaining, we will assume that the coalition chooses a Pareto efficient outcome for its members, and that the division of payoffs guarantees each party at least what they would get without collusion. For the last part of the paper we will further assume that the outcome of the negotiation between the manager and the auditor is the Nash bargaining solution.

Strategy spaces

The principal designs the contract and offers it to the manager and the auditors. He pays the manager $t(x)$ and uses the internal with probability $\gamma(x)$. For each θ the contract determines

an optimal level of effort (e_1 and e_2) yielding x_1 when $\theta = \theta_1$ and x_2 when $\theta = \theta_2$ [9]). He pays the internal $w(x,\hat{s}^i)$ and uses the external—at cost z—with probability $d(x,\hat{s}^i)$. $\hat{s}^i$ is the internal's report and it may be s_1, s_2 or 0 if the internal was not used.

If an auditor's announced signal differs from the manager's (implicit) report, the principal can impose a penalty on the manager of up to P^{m*}. If the external reports a signal different from the one of the internal, the principal can impose a penalty on the internal of up to P^{i*}. The feature of limited liability enables us to solve the model with risk-neutral players and obtain many of the features of a model with risk-averse players and unbounded punishments (see Sappington [1983] and Baron-Besanko [1984]).

The manager chooses effort e. He can collude with the internal to manipulate $\hat{s}^i$ in exchange of a side-payment $\tau(\hat{s}^i)$. More explicitly, $\tau(\cdot)$ should be written as a function of $\hat{s}^i$ given s^i However the manager has an interest in colluding only when $s^i = s_2$ (the high-productivity signal obtains). This is why $\tau(\cdot)$ is written as a function of $\hat{s}^i$ only. Finally, the external receives the signal s and reports it truthfully ($s^e = s$.) All players must be awarded their reservation utility level to sign the contract.

Timing

(1) Nature chooses the state of productivity (θ) and the signal for the auditors (s) according to Table 2.

(2) The manager learns θ. (He doesn't observe s yet.)

(3) The principal offers a contract specifying the transfer for the manager ($t(x,\hat{s}^i,\hat{s}^e)$) as a function of output and the auditors' reports, the transfer for the internal ($w(x,\hat{s}^i,\hat{s}^e)$) as a function of output, his report and the external's report, the (fixed) transfer for the

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9 If $x \notin \{x_1, x_2\}$, the agent gets punished, so we can restrict attention to outputs x_1, x_2.

external (z), the probabilities of conducting internal and external audits ($\gamma(x)$ and $d(x,\hat{s}^i)$) and the punishments for the manager and the internal (P^m and P^i). At the optimum all these functions will be depend on the parameters of the problem: the probability of a low type (q), the informativeness of the signal (r), the maximum punishment for the manager (P^{m*}), the maximum punishment for the internal (P^{i*}) and the cost of the external (z).

(4) The contract is signed. The manager chooses his effort (e). Output (x) is realized and observed by all parties.

(5) The internal is sent with probability $\gamma(x)$. The signal (s) is observed by the manager and the auditors.

(6) The manager and the internal can sign a side-contract specifying a transfer dependent on the internal's report ($\tau(\hat{s}^i)$).

(7) The internal reports $\hat{s}^i$. Side-transfers are realized.

(8) The principal asks for the external's report with probability $d(x,\hat{s}^i)$. The external reports $\hat{s}^e = s$.

(9) Transfers are realized.

	s_1	s_2
θ_1	q r	q (1-r)
θ_2	(1-q) (1-r)	(1-q) r

Table 2: Nature's Moves

The fact that the manager has to choose his level of effort before negotiating a side-contract with the internal greatly simplifies the bargaining problem. Indeed, they will bargain under symmetric information, both knowing the output x and the signal s. The fact that the

internal ignores the real state of productivity θ is irrelevant since the punishment can only depend on observed variables.

Note that we do not define the transfer as being net of punishments. When convicted of collusion, the manager gets the transfer corresponding to the state of productivity announced by the external minus the punishment and the internal gets the transfer corresponding to his report minus the punishment. This simplifies the presentation of the results without further consequences [10].

A remark must be made about the side-contract between the internal and the manager. Since this contract is usually illegal, it must be self-enforceable. We assume that there exists some mechanism [11] making the side contract self-enforceable. Finally, an objection can be raised to the validity of the contracts we will characterize since the resulting equilibrium strategies are not renegotiation-proof. We will consider the renegotiation problem in our conclusions and proposals for further research.

Preview of the Results

As a benchmark, we derive the first best contract and show that symmetric information is a necessary condition for its feasibility. In the first best scheme the manager is paid just enough to make him sign the contract and the marginal cost of compensating the manager

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10 With a limited liability interpretation, we should model the maximum punishment as a function of the transfer received by the manager. In our two-type model, however, the punishment is only applied to the low type. Therefore, it is correct to consider a unique level of maximum punishment.

11 For example, the firm could pay the auditor with "half dollar bills" before he submits his audit report. Once the report specified in the side-contract has been filed, the firm has no interest to keep the other halves of the dollar bills. Making a deposit on an account with 2 signatures (auditor and manager) would be a similar device. Reputation could be another alternative to make the side-contract enforceable.

for increasing his effort is equated to the marginal benefit for the principal of increasing the manager's effort.

When the principal cannot observe the manager's effort the first best contract is not feasible because the manager's incentive compatibility constraint is not satisfied: if the principal pays the manager a fixed amount, the high productivity manager will shirk. A standard result in contract theory is that the second best scheme will induce an inefficient level of effort from the low type as a means of achieving incentive compatibility with the least cost; a small reduction in the low type's effort involves a second-order loss in profits while the relaxation of the incentive compatibility constraint generates a first-order gain (see Baron-Myerson [1982] and Maskin-Riley [1984]). On the other hand, the high type exerts the first best level of effort (this result is known in the literature as "no inefficiency at the top"). Finally, the low type does not get any rent (his individual rationality constraint is binding) while the high type does (his individual rationality constraint is slack). (See proposition 1.)

Next we introduce a faithful and costless auditor. He receives a signal imperfectly correlated with the state of the world and reports it to the principal. We show that according to the relation between the accuracy of the auditor's signal and the maximum liability of the manager, the effect of the auditor can be separated in three regions. In the first region, the principal reduces the informational rent of the high type. In the second region, the rent has been reduced to zero and the effort of the low type is adjusted towards the first best. Finally if the information of the auditor is sufficiently accurate with respect to the available punishment, the first best (third region) is reached. (See proposition 2.)

Next we let the faithful auditor be costly. Compared to the scheme with free auditing, two new regions appear. In the first one, the principal does not use the auditor because he is too expensive relative to his information and the maximum punishment available. In the second one, the principal controls the probability of sending the auditor. We show that it is best to

decrease the probability of audit while the effort of the low type is still suboptimal. The first best can still be reached, but only at the limit when the punishment grows without bound. (See proposition 3.)

Next we allow the manager and the auditor to sign a side-contract after the manager chose his level of effort and both he and the auditor (jointly) observe the signal. We show that the first best is never reached in this model (as long as the auditor's information is not perfect; i.e., $r < 1$). The reason is that, at some point, the principal does not want to use all his punishment capacity, i.e., (expected) maximum deterrence is not optimal. The intuition is that high punishments induce high bribes which are costly to deter. (See proposition 4.)

The next step is to introduce a second auditor. To model the trade-offs involved in the achievement of coalition incentive compatibility, we assume that this new auditor always reports truthfully but is costly. While the inside auditor is inexpensive and can be bribed, the external is expensive but cannot be bribed. The optimal contract specifies four main regions depending on the values of r and P^m. For low values of r and P^m no auditor is used. In the second region, the values of r and P^m are such that the internal auditor is always used, but the external is called to audit on a random basis (the probability of external audit is strictly between zero and one.) In the third region, r and P^m are high enough for the optimal coalition-free contract to use only the internal. When $r = 1$ and P^m is high enough the first best is achieved. (See proposition 5.)

Finally, we extend the model to consider auditors of different "types." We let the internal auditor be randomly drawn from a distribution which has truthful (as we have been assuming the external to be so far) and completely self-interested (as we have been assuming the internal to be so far) types. In this case, we claim that for some range of the parameter space, the principal will find it optimal to allow collusion (See proposition 6) This non-screening mechanism produces a more realistic account of empirical observations: some people do not collude and do not get caught, some people do not collude and get

punished by mistake, some people collude and do not get caught, some people collude and get punished.[12]

First Best Allocation

Consider the case in which output and effort exerted by the manager are observable by the principal [13]. Auditors have no role and a contract which specifies the manager's transfer as a function of effort and the state of productivity is feasible. The optimal level of effort maximizes the profit minus the cost of inducing the manager to take that effort.

The principal's problem is to choose e_1, e_2, t_1, t_2 to

$$\text{Max}\; q\,\{\theta_1 + e_1 - t_1\} + (1 - q)\,\{\theta_2 + e_2 - t_2\}$$

subject to:

(MIR1) $\quad t_1 \geq \frac{e_1^2}{2}$

(MIR2) $\quad t_2 \geq \frac{e_2^2}{2}$

The solution of the above problem has $e_1 = e_2 = e^{FB} = 1$ and $t_1 = t_2 = t^{FB} = 1/2$.

In the optimal contract under symmetric information, the principal pays the agent just enough to make him sign the contract; i.e., $e^2/2 = 1/2$ and equates the marginal cost of the agent's effort to its marginal value product. Also note that the transfer received by the agent is independent of the state of productivity; $t(x_1) = t(x_2) = 1/2$.

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12 For future research and preliminary speculate about a mode where two auditors of unknown type are available to the principal. Only with a second auditor collusion may be uncovered and punished.

13 And therefore, the state of productivity can be inferred with certainty.

Graphically, we can represent the agent's indifference curves in the product-transfer space and find the solution where the slope of these curves is 1.

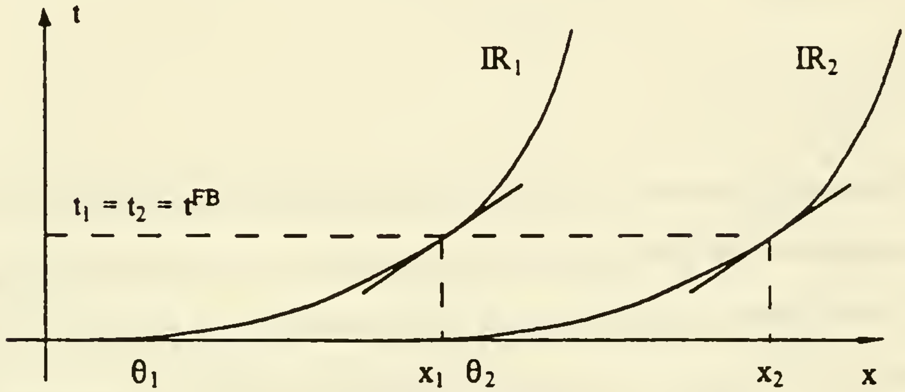

Figure 1. First Best Scheme

No Auditor

From now on, we will assume that the principal can neither observe the state of nature nor the manager's effort. Thus, the contract can only specify transfers as a function of the output (x). If the principal tried to apply the first best scheme described above, the manager would always produce x_1.

Under these circumstances, the problem (which we label B0) for the principal is to choose e_1, e_2, t_1, and t_2 to

$$\text{Max } q\{\theta_1 + e_1 - t_1\} + (1-q)\{\theta_2 + e_2 - t_2\}$$

subject to

(MIR1) $\quad t_1 \geq \frac{e_1^2}{2}$

(MIR2) $\quad t_2 \geq \frac{e_2^2}{2}$

(MIC1) $$t_2 - \frac{e_2^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2}$$

(MIC2) $$t_1 - \frac{e_1^2}{2} \geq t_2 - \frac{(e_2 - \Delta\theta)^2}{2}$$

where $\Delta\theta = \theta_2 - \theta_1$.

The manager's individual rationality constraints (MIR) state that he must be at least compensated for the cost of his effort in each state of the world. Our implicit assumptions are that the manager signs the contract after learning θ and that his reservation utility is normalized to be zero. The manager's incentive compatibility constraints (MIC) appear because the principal can not differentiate between states of nature one and two, therefore some incentives must be provided to the manager if he is to exert the effort desired by the principal.

As usual, only MIR1 and MIC2 are binding at the optimum. It can indeed be verified ex post that the other two constraints are satisfied at the optimum when they are ignored in the optimization program.

Graphically, the solution is:

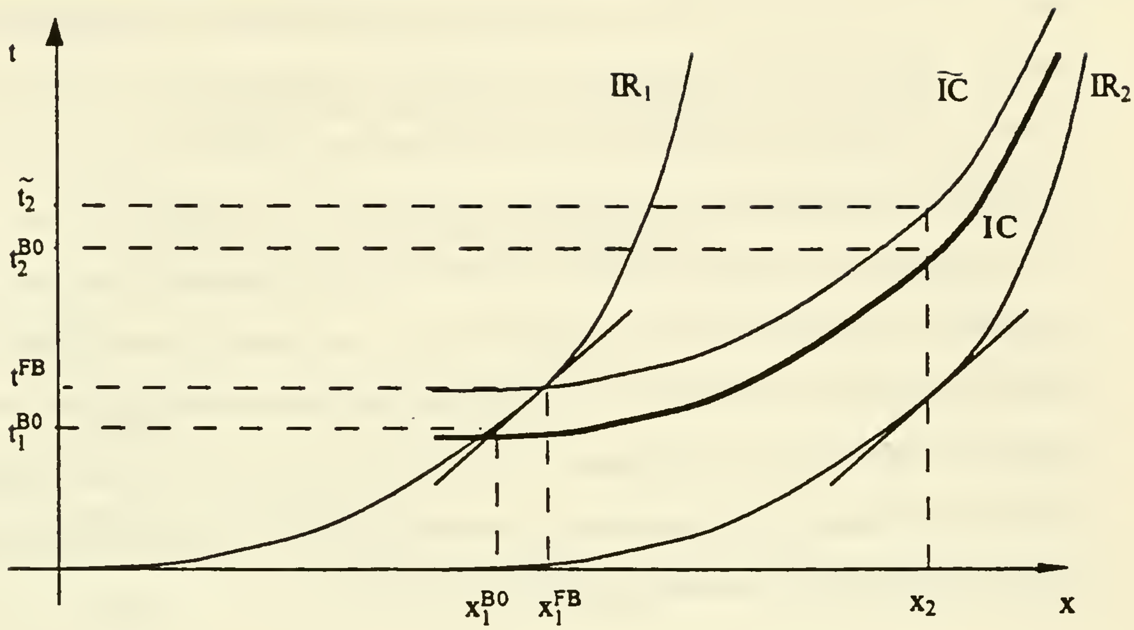

Figure 2. Optimal Contract with No Auditor

Proposition 1 (B0 Scheme): In the absence of auditors, the optimal contract between the principal and the manager specifies:

$$e_1 = 1 - \frac{1-q}{q}\Delta\theta < 1$$

$$e_2 = 1$$

$$t_1 = \frac{e_1^2}{2} = \frac{1}{2} - \left(\frac{1-q}{q}\right)\Delta\theta + \frac{1}{2}\left(\frac{1-q}{q}\right)^2 \Delta\theta^2$$

$$t_1 = \frac{1}{2} + \frac{e_1^2}{2} - \frac{(e_1 - \Delta\theta)^2}{2}$$

Proof: The proof is standard and, therefore, omitted.

When the principal cannot observe the manager's effort the first best contract is not feasible because it does not satisfy the MIC constraint. If the principal pays the manager a fixed amount, a highly productive manager will always shirk. A standard result in contract theory is that the second best scheme will induce an inefficient level of effort from the low type θ_1 as a means of achieving incentive compatibility with the least cost: a small reduction in type 1's effort involves a second-order loss in profits while the relaxation of the incentive compatibility constraint generates a first-order gain (see Baron-Myerson [1982] and Maskin-Riley [1984]). On the other hand, the high type θ_2 exerts the first best level of effort (this result is known in the literature as "no inefficiency at the top"). Finally, the low type does not obtain any rent (his individual rationality constraint is binding) while the high type does (his individual rationality constraint is slack.)

One Free Faithful Auditor

The principal can now use an auditor who observes a signal s (imperfectly) correlated with the true state of productivity θ, and reports it truthfully ($\hat{s} = s$). We normalize the auditor's reservation salary to zero and assume that his wage can be conditioned contractually on the manager's output and his report.

If the principal receives a report indicating that the manager shirked

$$\hat{s} \neq x_i - e_i \qquad (i = 1,2)$$

he can impose a penalty on the manager of P^m bounded above by P^{m*}. Baron and Besanko [1984] have shown that the principle of maximum deterrence holds in this model [14], hence, we will consider from now on that $P^m = P^{m*}$. It is also easy to check that there is

14 For the general result of maximum deterrence in the presence of mistakes see Bolton [1987].

no need to use the auditor when the high output x_2 is realized and that the auditor's salary will not exceed his reservation value of zero.

The problem faced by the principal now [15] is to choose e_1, e_2, t_1 and t_2 to

$$\text{Max } q\ \{\theta_1 + e_1 - t_1 + (1 - r)\ P^m\} + (1 - q)\ \{\theta_2 + e_2 - t_2\}$$

subject to:

(MIR1) $$t_1 \geq \frac{e_1^2}{2} + (1 - r)\ P^m$$

(MIR2) $$t_2 \geq \frac{e_2^2}{2}$$

(MIC1) $$t_2 - \frac{e_2^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2} - r\ P^m$$

15 Notice that he risk neutrality of the agent allows us to state MIR1 as a lower bound for his expected transfer.

Graphically, the solution is:

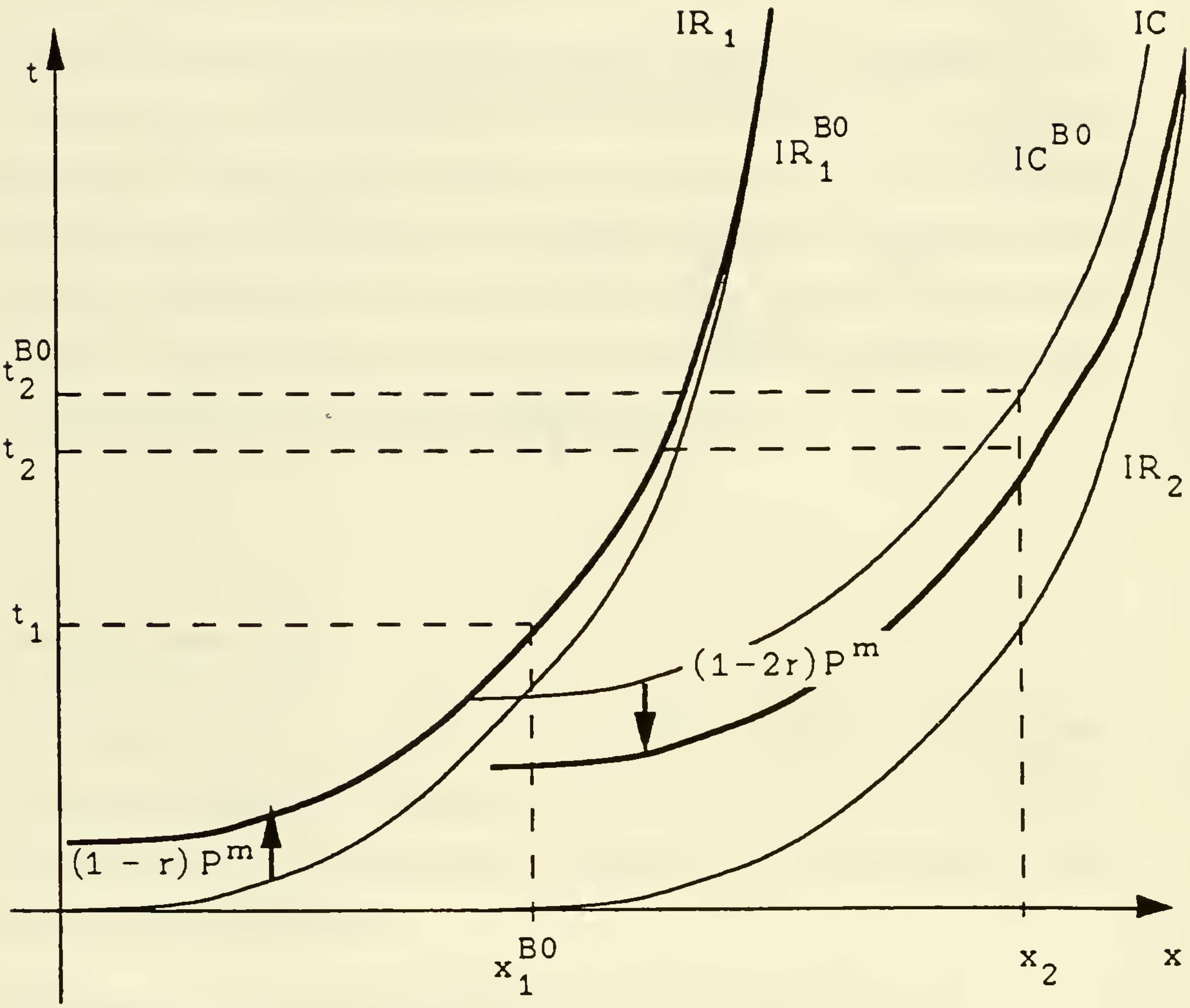

Figure 3. Optimal Contract with a Free Faithful Auditor

Proposition 2: The optimal contract with a free faithful auditor can be divided in three regions according to the values of r and P^m.

(B1) If $P^m < \frac{\Delta\theta}{2r-1}\left[1 - \frac{\Delta\theta}{2q}\left(2 - q\right)\right]$, (MIR2) is slack and the difference with the scheme (B0) is that $t_2 = t_2^{B0} - (2r-1)P^m$ (the rent is reduced)

(B2) If $\frac{\Delta\theta}{2r-1}\left[1 - \frac{\Delta\theta}{2q}\left(2 - q\right)\right] < P^m < \frac{\Delta\theta}{2r-1}\left[1 - \frac{\Delta\theta}{2}\right]$, (MIR2) binds and the differences with the scheme (B1) are that

$e_1 = \frac{2r-1}{\Delta\theta} P^m + \frac{\Delta\theta}{2}$ (the effort is adjusted)

$t_2 = \frac{e_2^2}{2} = \frac{1}{2}$ (the rent is zero)

(FB) If $P^m \geq \frac{\Delta\theta}{2r-1}\left[1 - \frac{\Delta\theta}{2}\right]$ the first best is achieved.

Proof: The proof is standard and, therefore, omitted.

The mechanism can be described sequentially as follows: the principal observes the output produced by the manager and gives him the corresponding transfer. If output is high, there is no further action. If output is low the principal requires the report of the auditor. If the auditor reports a 'high' signal the manager is punished with P^m.

In the first region, with low accuracy and small punishments, only informational rents are decreased while effort levels for each type of manager remain unchanged. This is

analogous to the Baron-Besanko's separation result which stated that the output decision is made independently of the auditing decision (although the auditing decision is not made independently of the output decision). In other words, the effort exerted by each type of manager in the optimal contract with auditing is the same one he exerted in the optimal contract without auditing. As the accuracy of the auditor's signal or the maximum punishment increases, the rent of the high type manager decreases but the effort of the low type manager is not adjusted.

In the second region, the rent of the high type has been reduced to zero because the information of the auditor is sufficiently accurate relative to the available punishment. The individual rationality constraint of the high type is now binding. The principal can therefore reduce the inefficiency imposed on the low type's effort and still achieve incentive compatibility. If the information improves or the punishment is increased even more, the distortion disappears and the first best is reached. Another (more intuitive) consequence of this result is that infinite punishments can support the first best with a signal that is arbitrarily close to non-informative (r ξ 1/2 in our case).

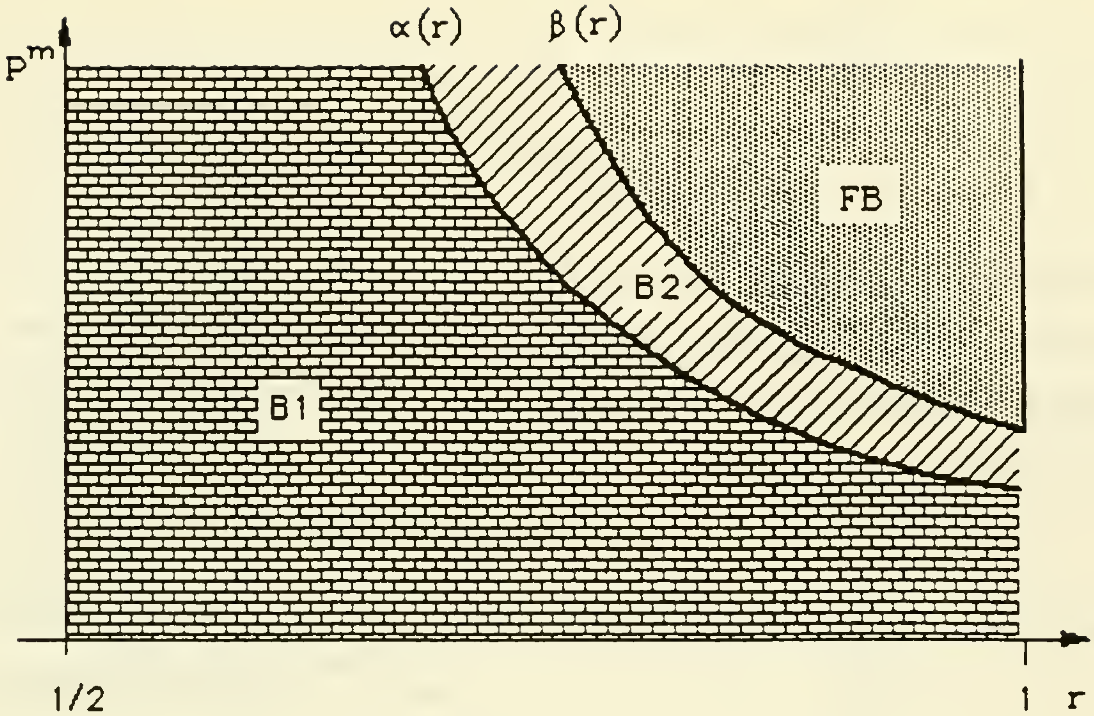

Figure 4. Regions of the Optimal Contract with a Free Faithful Auditor

The following graphs are useful to see the change in the principal's profits, the low type's effort and the high type's rent:

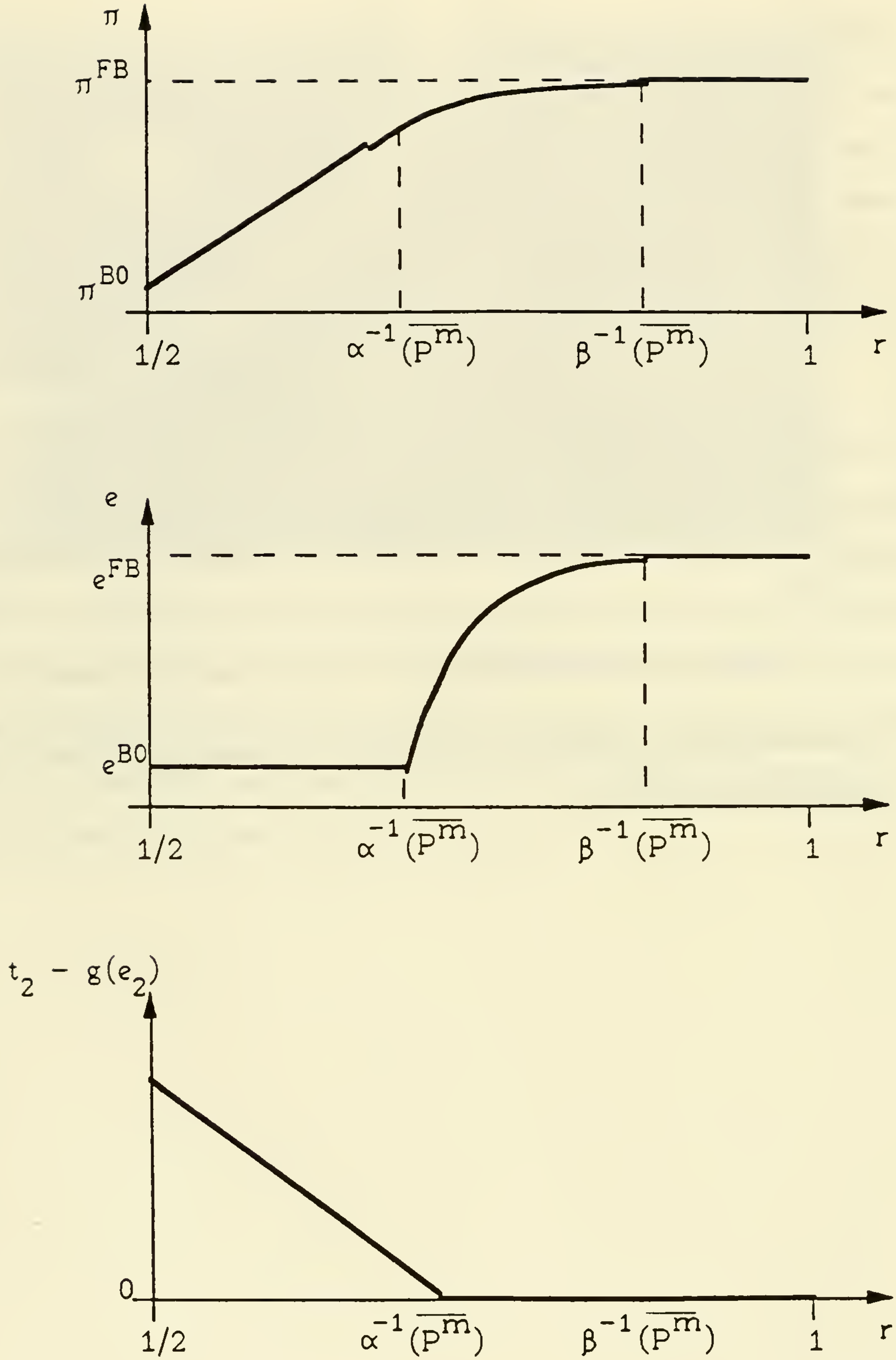

Figure 5. Profits, Effort and Rent in the Optimal Contract
with a Free Faithful Auditor

One Costly Faithful Auditor

Normalizing the auditor's reservation wage to zero was not without loss of generality. Two new effects come into play when the auditor is costly. First, the principal might decide not to audit at all because the auditor is too expensive. Second, if the maximum available punishment is high enough, it will be optimal to reduce the probability of auditing before the effort of the low type is restored to its first best level.

Suppose the auditor costs z. The principal's problem is to choose e_1, e_2, t_1, t_2, and γ (the probability of auditing) to

$$\text{Max}\; q\,\{\theta_1 + e_1 - t_1 + \gamma\,[(1-r)\,P^m - z]\} + (1-q)\,\{\theta_2 + e_2 - t_2\}$$

subject to:

(MIR1) $\quad t_1 \geq \frac{e_1^2}{2} + \gamma\,(1-r)\,P^m$

(MIR2) $\quad t_2 \geq \frac{e_2^2}{2}$

(MIC) $\quad t_2 - \frac{e_2^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2} - \gamma\, r\, P^m$

Define:

$\kappa(r,z)$, the minimum value of P^m such that it is profitable to use an auditor costing z and having information r

$$\kappa(r,z) = \frac{z\,q}{(1-q)\,(2r-1)}$$

$\omega(r,z)$, the value of P^m at which the benefit of adjusting effort is equal to the benefit of reducing the probability of sending the auditor (γz), is implicitly given by

$$2\, P^{m2} (2r - 1)^2 + P^m \Delta\theta\, (2r - 1)\, (\Delta\theta - 2) + 2\, z\, \Delta\theta^2$$

which implies:

$$\omega(r,z) = \frac{\Delta\theta \left[(2 - \Delta\theta) + \sqrt{(2 - \Delta\theta)^2 - 16\, z} \right]}{4\, (2r - 1)}$$

Notice that $\omega_z < 0$.

Proposition 3: The optimal contract can be divided in four regions according to the values of r, P^m and z.

(B0) If $(2r - 1) \frac{1 - q}{q} P^m \leq z$, the auditor is not used ($\gamma = 0$) and scheme B0 is applied.

If $(2r - 1) \frac{1 - q}{q} P^m \geq z$, the auditor is used.($\gamma > 0$)

(B1) If $P^m \leq \frac{\Delta\theta}{2r - 1} \left[1 - \frac{\Delta\theta}{2q} \left(2 - q \right) \right]$; $\gamma = 1$, (MIR2) is slack and scheme B1 is applied.

(B2) If $\frac{\Delta\theta}{2r - 1} \left[1 - \frac{\Delta\theta}{2q} \left(2 - q \right) \right] < P^m \leq \omega(r,z)$; $\gamma = 1$, (MIR2) binds and scheme B2 is applied.

(E3) If $P^m > \omega(r,z)$; $\gamma = \frac{\Delta\theta\, [(2 - \Delta\theta)\, Pm\, (2r - 1) - 2\, z\, \Delta\theta}{2\, P^{m2}\, (2r - 1)^2} < 1$ and

$$e_1 = 1 - \frac{z\, \Delta\theta}{P^m \left(2r - 1 \right)}$$

This we call the scheme E3 (where E refers to "external").

(FB) When $P^m \to \infty$, the first best is approximated arbitrarily closely.

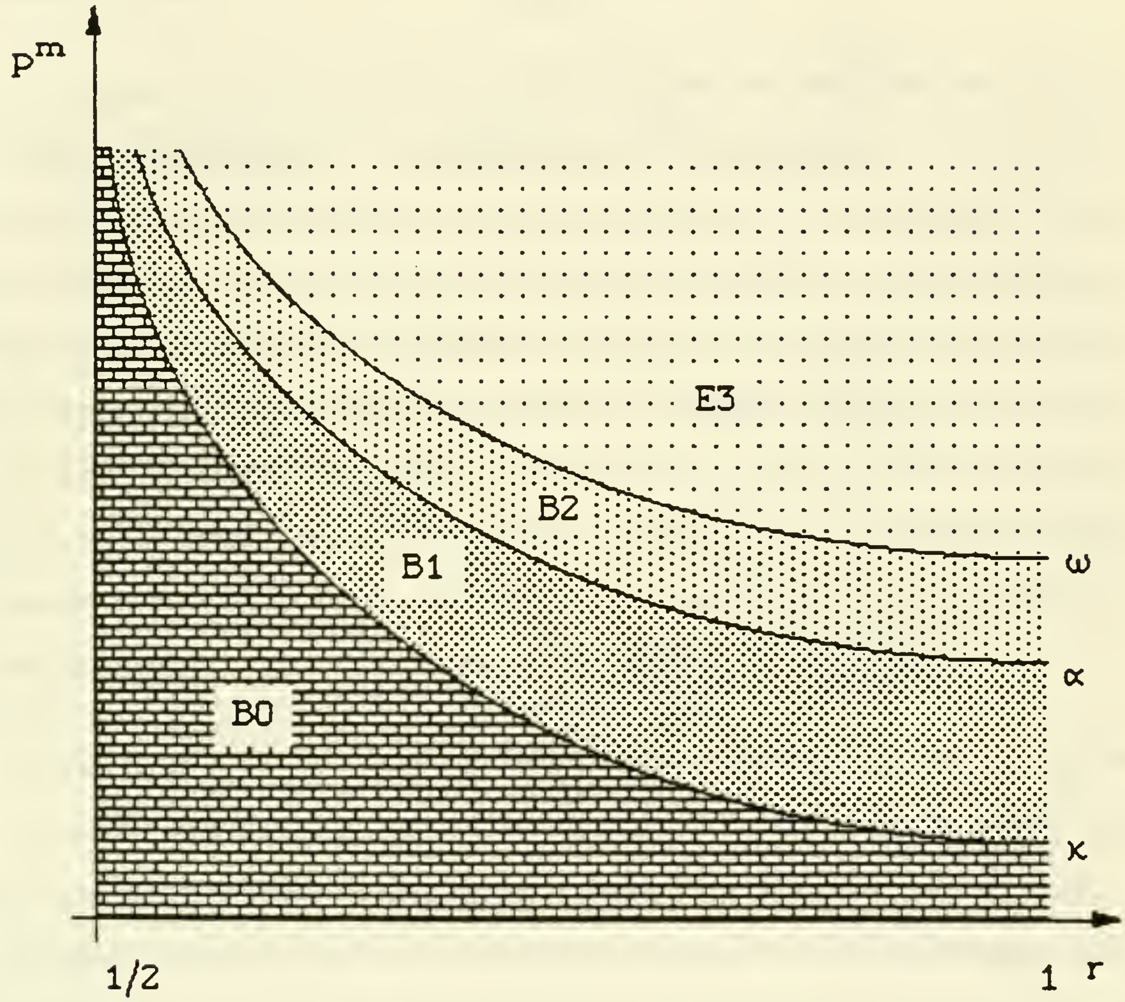

Figure 6. Regios of the Optimal contract with a Costly Faithful Auditor

We can observe again how the principal's profits, low type's effort and audit probability change with r:

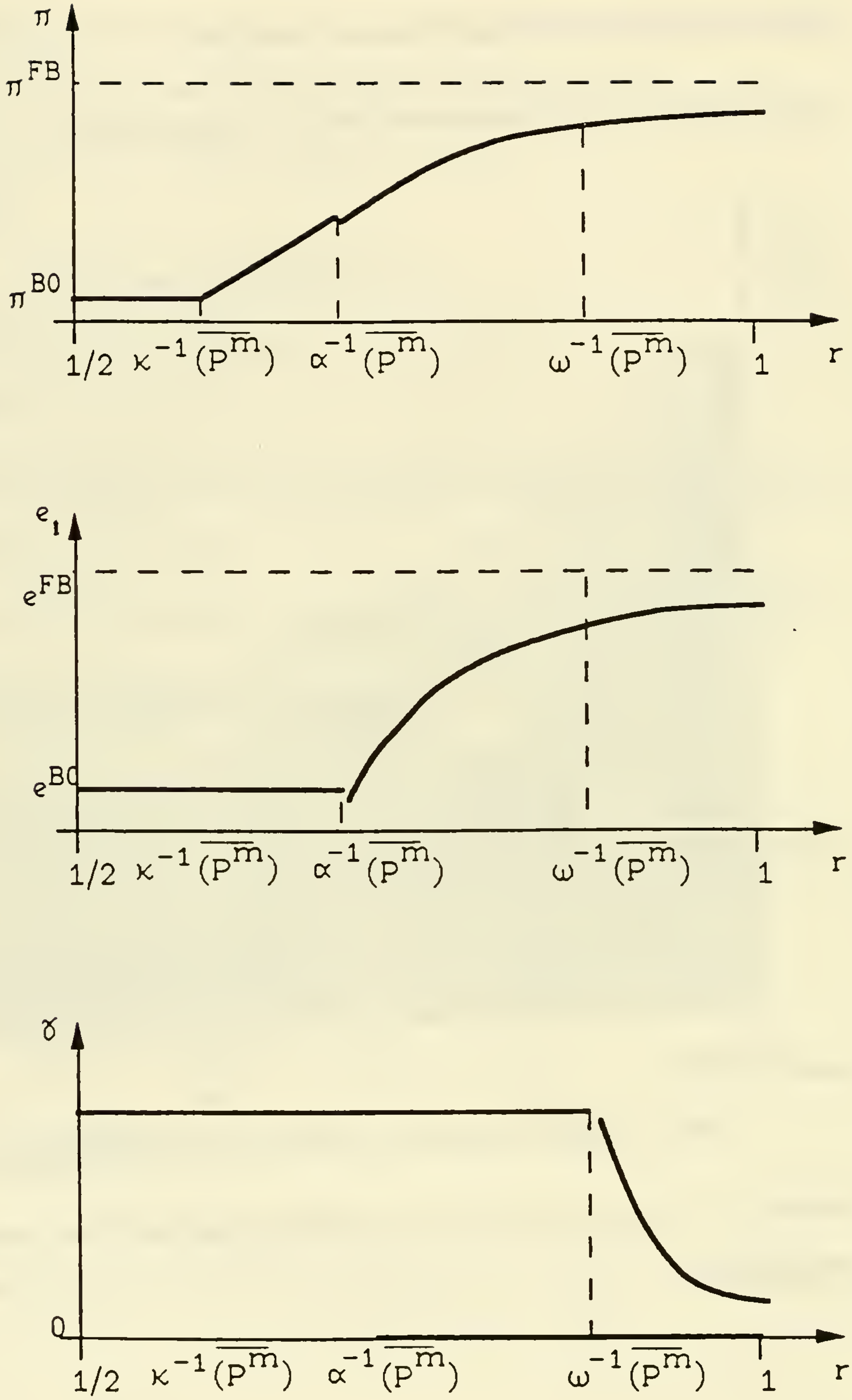

Figure 7. Profit, Low Type's Effort and Audit Probability in the Optimal Contract with a Costly Faithful Auditor

Compared to the scheme of proposition 2, two new regions appear. First, in region B0 the principal does not use the auditor because he is too expensive relative to his information and the maximum punishment available. If the cost of the auditor increases, the region B0 increases as well.

If the principal does use the auditor, the optimal scheme is very similar to the one presented in proposition 2. In region B1, the principal extracts rent of the high type. If the maximum punishment and the informativeness of the auditor become sufficiently high, all the informational rent of the high type can be extracted and the effort of the low type can be adjusted towards the first best level. However, and this is the second major difference with proposition 2, it is optimal to decrease the probability of audit (γ) while the effort of the low type is still suboptimal. The first best is only reached at the limit when the punishment grows without bound. The trade-off driving this result has, on the one hand, the possible increase in profits due to adjusting the effort of the low type and, on the other hand, the possible increase in profits due to decreasing the probability of a costly audit.

Consider, for instance, the case in which the maximum punishment increases. In region B1, when P^m is increasing, the principal can choose between decreasing the probability γ and extracting more rent from the high type. Lemma 3.1 proves that it is never optimal to decrease γ. The reason is that in order to make the reduction of γ profitable compared to the rent extraction, the cost of auditing would have to be so large that it would not be optimal to use the auditor at all.

So if γ becomes less than one, it has to be in region B2. Indeed, lemma 3.2 shows that for any r, there exists a P^m for which $\gamma < 1$ even though the effort level of the low type is below the first best. This means that at some point the principal becomes indifferent (at the margin) between decreasing γ and adjusting the effort. This is where the region E3 starts. The line separating B2 and E3 is called $\omega(r,z)$. Along $\omega(r,z)$ and in region E3, the marginal

profit of decreasing the probability of auditing equates the marginal profit of restoring the effort.

At this point, it is useful to highlight two facts: (1) The convexity of the effort function makes the profit function concave in P^m via effort adjustment. Hence, in region B2 effort adjustment becomes less and less profitable while approaching its optimal level (at the first best, the benefit of adjusting effort is zero.) (2) The profit from lowering the probability of auditing is a concave (increasing) function of the available maximum punishment (an inverted hyperbola). To see this, notice that it is the *expected* punishment which affects the manager's incentives to tell the truth. So if the level of punishment increases, the expected punishment can be kept constant by decreasing the probability of audit at an inversely proportional rate.

Therefore, above line $\omega(r,z)$ (i.e., in region E3) the principal will simultaneously decrease the probability of auditing and increase the effort for the low type so that he equates the marginal profit of those two actions. Finally, if the punishment is allowed to grow without bound, the optimal contract goes to the fist best level of effort with an arbitrarily low probability of audit.

Notice that the punishment capacity will be fully exercised in the optimal contract; it is strongly optimal to use maximum deterrence (of course, whenever the auditor is used at all). The decision of the principal is when to use this punishment to extract rents, when to adjust effort and when to reduce the probability of auditing.

One Free Unfaithful Auditor

Next we allow the manager and the auditor to collude signing a side-contract after the manager chose his level of effort and both him and the auditor (jointly) observed the signal. It is readily checked that the only case for collusion is when the productivity of the manager

is low and the signal of the auditor is high. If the manager has high productivity (and produces high output) the principal does not use the report of the auditor. If the manager has low productivity and the auditor receives a low signal there is no reason for a side-transfer since the auditor cannot forge evidence by himself.

According to the audit technology we have specified and the timing of the game, whether the signal is right or wrong is irrelevant for studying the coalition. At the moment of the negotiation, the manager has already chosen his effort and regardless of the true state of the world he knows that even with truthful reporting he will be punished if a high productivity signal obtains. And, since the optimal contract is incentive compatible, the manager never shirks. He is only punished by mistake and everybody knows it [16]. To avoid the punishment P^m the manager is willing to pay the auditor up to P^m. Thus, in the presence of a covert contract, the auditor would not report payoff-relevant information and the allocation described in section above is no longer sustainable. The auditor would be indifferent between releasing or not information leading to the punishment of the manager, but the manager is ready to "buy the auditor's silence".

This gives rise to a new constraint, the coalition incentive compatibility (CIC) constraint. To induce truth-telling when the auditor has evidence to punish the manager, the principal must pay him at least as much as the manager would pay him to change his report; i.e., the auditor must be rewarded with at least the amount by which the manager would be punished if the auditor revealed his incriminating information.

Also, as stated above, we assume that the manager and the auditor choose a side-contract that is Pareto efficient and individually rational with respect to the no side-contract outcome.

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16 This is a standard result from game theory. For a similar result see Green and Porter [1984].

The problem for the principal now is to choose e_1, e_2, t_1, t_2 and w to:

$$\text{Max } q\,\{\theta_1 + e_1 - t_1 + \gamma\,(1 - r)\,(P^m - w)\} + (1 - q)\,\{\theta_2 + e_2 - t_2\}$$

subject to:

(MIR1) $$t_1 \geq \frac{e_1^2}{2} + \gamma\,(1 - r)\,P^m$$

(MIR2) $$t_2 \geq \frac{e_2^2}{2}$$

(MIC) $$t_2 - \frac{e_2^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2} - \gamma\, r\, P^m$$

(CIC) $$w \geq P^m$$

Where γ is the probability of using the auditor, and w is his salary in the event of exposing the manager [17].

Two new features can be observed in the constraints: the presence of γ and the coalition incentive compatibility constraint (CIC). Since the auditor colludes, it may be optimal for the principal not to send him ($\gamma = 0$); if truthful reporting is to be achieved, CIC must be fulfilled.

We have implicitly ruled out the possibility that the auditor pay up-front the expected reward he will get by exposing the manager [18]. If the principal is allowed to sell the rights of auditing, the auditor will only receive his reservation utility. The principal will then be in a situation analogous to the one discussed in the model with a faithful auditor.

Define:

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17 It is clear that the auditor will receive zero when he reports low productivity ($\hat{s}_1 = s_1$).

18 This possibility would not be optimal if the auditor was infinitely risk averse or had a liquidity constraint.

$$\alpha = \frac{2\,q\,\Delta\theta + q\,\Delta\theta^2 - 2\,\Delta\theta^2}{2\,q\,(2r - 1)}$$

as the value of P^m at which MIR2 becomes binding and

$$\beta = \frac{\Delta\theta\,(4r - \Delta\theta - 2)}{2\,q\,(2r - 1)^2}$$

as the value of P^m at which the benefit of adjusting effort is equal to the cost of increasing the expected punishment (γP^m).

Proposition 4: The optimal contract can be divided in five regions according to the values of r and P^m.

(B0) If $r \leq \frac{1}{2 - q}$, the auditor is not used ($\gamma = 0$) and scheme B0 is applied.

If $r > \frac{1}{2 - q}$, the auditor is used ($\gamma > 0$).

(B1) If $1 > r > \frac{1}{2 - q}$ and $P^m \leq \alpha(r)$, scheme B1 is applied.

(B2) If $1 > r > \frac{1}{2 - q}$ and $\alpha(r) \leq P^m < \beta(r)$ scheme B2 is applied.

(B3) If $1 > r > \frac{1}{2 - q}$ and $\beta(r) < P^m$ scheme B3 is applied.

(FB) If $r = 1$ and $\beta(r) < P^m$ the first best is attained.

Definition of scheme B3:

$$\gamma(r, P^m) = \frac{\beta(r)}{P^m} \quad ; \quad w = P^m \quad ; \quad e_1^{B3}(r) = 1 - \frac{\Delta\theta\,(1 - r)}{2r - 1}$$

Proof: see appendix

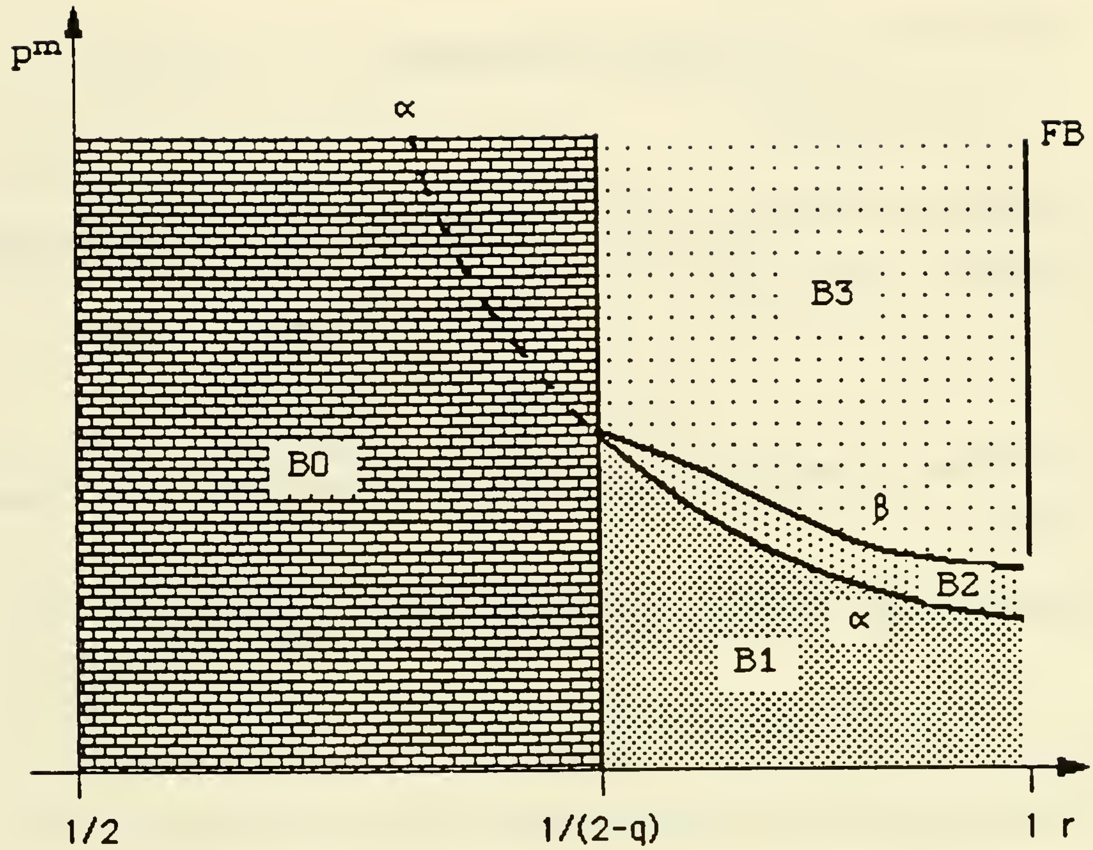

Figure 8. Optimal Contract with a Free Unfaithful Auditor

We can plot the evolution of the principal's profits and the probability of an audit in the case where $\frac{1}{2-q} < r < 1$:

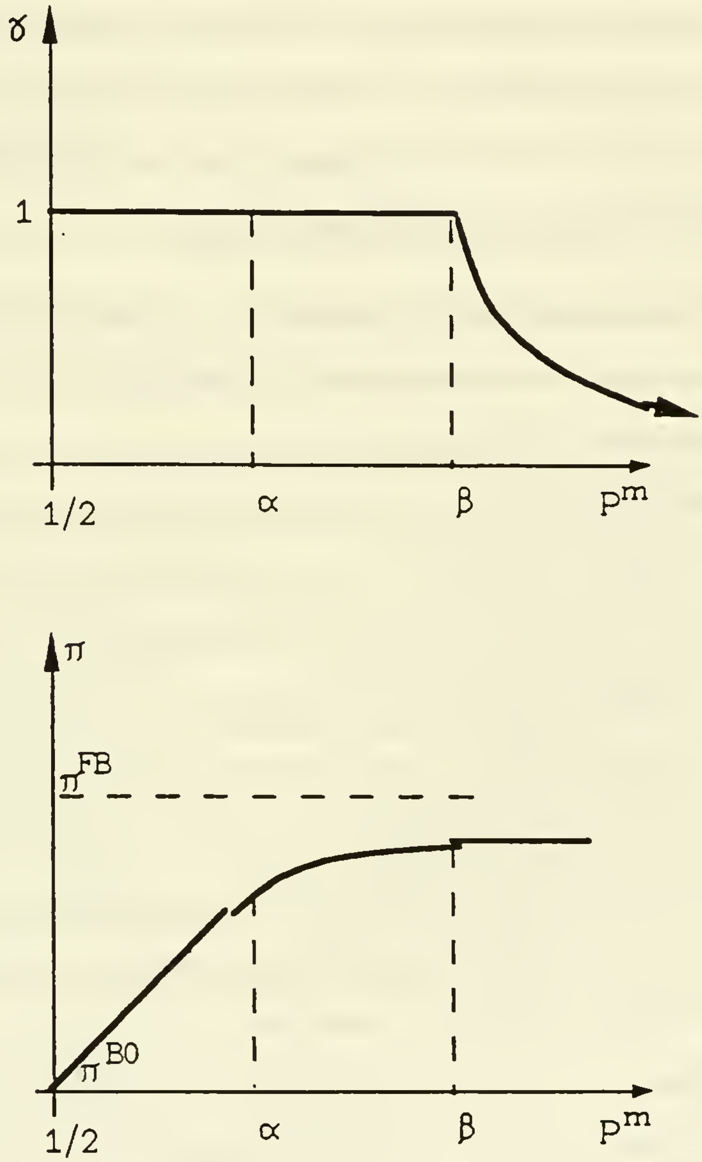

Figure 9. Probability of Audit and Profit when the Auditor is Used

If the signal is too noisy—i.e., r is small ($r \le (1/2\text{-}q)$) the auditor is not used (region B0). It is not optimal to use the auditor even though his reservation wage is zero because this time it is costly to induce truthful revelation. The cost comes from compensating the manager for mistaken punishments. To understand why the critical value of r depends on

q, remember that our optimal mechanism is collusion free and incentive compatible. Therefore, the principal has to pay the auditor's reward w only when the auditor makes a mistake (which happens with probability 1-r) after the low productivity type obtained (which happens with probability q). To justify the use of an auditor, the higher the proportion of low type is, the better the information must be. When the auditor not is used, the incentive scheme is similar to the one in proposition 1.

If the auditor's signal is good enough with respect to the probability of low productivity ($r > 1/(2-q)$), the principal rewards the auditor to discourage him from colluding with the manager and the contract is similar to the one of proposition 2. A new separation result obtains in regions B1 (where the rent is extracted) and B2 (where the effort of the low type is restored). Compared to proposition 2, the profits are however lower because the principal has to pay a reward w to the auditor with probability $q(1-r)$ to prevent collusion. Another major difference with the scheme of proposition 2, is that the first best is never reached for an imperfect signal ($r < 1$) even if the maximum liability grows without bound. The intuition for this result is that increasing the manager's expected punishment has two effects: first, it allows the principal to achieve incentive compatibility from the manager at a lower cost but, at the same time, it increases the cost of getting truthful revelation from the auditor. Indeed, when P^m is high, the potential bribe to the auditor will be high as well, making the fulfillment of the CIC constraint expensive. At some point, determined by line $\beta(r)$, the second effect outweighs the first and the principal does not want to increase the expected punishment anymore while production is bounded away from the first best.

A major difference with the scheme E3 in proposition 3 is that the effort is never adjusted in region B3. The reasoning is as follows. In this model, the product γP^m must be kept constant in order to induce incentive compatibility. Recall that the cost of the auditor for the principal is γP^m (weighted by 1-r). Suppose that at some point when P^m increases, the principal finds profitable to decrease γ instead of adjusting the effort. Once γ and P^m have

been adjusted, the product (γP^m) is still the same as before (by MIC). So if it was profitable not to increase γP^m before, it has to be the case that it is still not profitable to do so now. Compared with the scheme E3 in proposition 3, the major difference comes from the fact that the cost of the auditor varies proportionately with P^m in proposition 4 but not in proposition 3 (the cost is z).

Then the punishment for the manager, the effort of the low productivity type and the profit of the principal are kept constant even if the maximum punishment available increases. In other words, even if higher punishments are available it is not optimal to use them.

So, even though production does not reach its first best level, *expected* maximum deterrence is not optimal. In this model, we can also claim that maximum deterrence is not optimal either. However this result comes from the fact that the auditor's reservation wage is zero. Each time P^m appears in our maximization problem, it is multiplied by γ, the probability of sending the auditor. If the auditor was costly, it would be optimal to decrease γ and always use maximum deterrence above the line $\beta(r)$. In our model, the principal is indifferent between decreasing γ or not increasing P^m. What really matters here is the product γP^m. This is why in proposition 4 the B3 scheme is with $P^m = P^{m*}$ and $\gamma < 1$. Alternatively P^m could be replaced by $\beta(r) < P^m$ and γ would be always equal to 1.

Two puzzling features appear in this model: the auditor is earning some rent and the first best is not attainable with unbounded punishments even though all players are risk neutral. These results depend crucially on the fact that we do not allow the principal to "sell" the right to audit, or to impose on the auditor negative transfers when he reports that the manager complied. Either possibility would take us back to the case of a faithful auditor (see footnote 17).

The following graph is an illustration of the profit-loss caused by collusion when P^m is such that it all regions are crossed when r goes from 1/2 to 1. i.e.,

$$\frac{2-q}{q}(\Delta\theta - \frac{\Delta\theta^2}{2}) \geq P^m \geq \Delta\theta - \frac{\Delta\theta^2}{2}$$ [19].

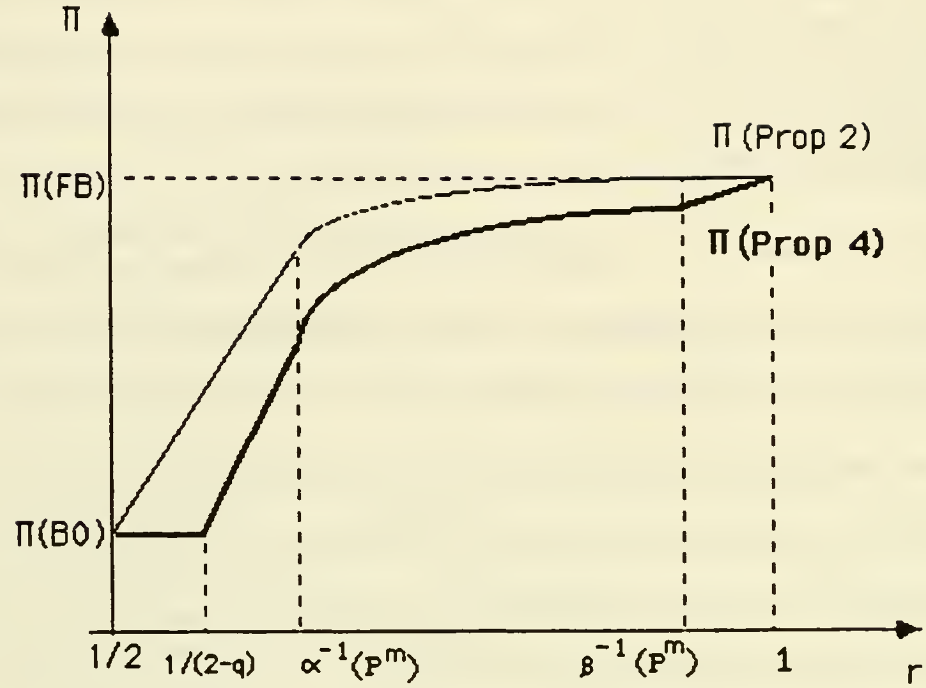

Figure 10. Profits as a function of r

Self-Interested Internal, Truthful External

Usefulness of a second auditor

Suppose that 2 signals are available and have the same cost [20] but the first one is more informative ($r_1 > r_2 > 1/2$). Suppose the principal gets the first signal saying that the manager shirked and should be punished. If the second signal confirms the first, the

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19 $(\frac{2-q}{q})(\Delta\theta - \frac{\Delta\theta^2}{2})$ is the P^m where $\alpha(r)$ and $\beta(r)$ cross. $(\Delta\theta - \frac{\Delta\theta 2}{2})$ is the minimum P^m necessary to achieve the first best when $r = 1$.

20 If the cost and informativeness are different, the principal should consider the informativeness per dollar.

principal is "more certain" that the manager shirked. If the second signal contradicts the first, he is "less certain" but he would still punish the manager because the probability that he shirked is higher than the probability that he did not.

The upshot of the argument is that in no state of the world the decision prior to the second signal is altered by it. Therefore, the second signal is worthless. If two signals (auditors) are available, the optimal contract will only consider one of them even if they are not correlated. If the two signals are equally informative, the least costly will be used, if they are equally costly, the most informative will be chosen [21].

This result is a very particular feature of our model. We do not think it is realistic, but it will later enable us to highlight the specific role of the external auditor as 'policing the police.' Since the external adds no value for the principal when the internal does not collude the only reason for the principal to pay for his services is to audit the internal.

Optimal Contract

Suppose now that the principal has the additional opportunity of hiring an external auditor at a cost $z > 0$. The two auditors get the same signal imperfectly correlated with the type of the manager (θ) according to table 1.

We are interested in the tradeoff involved in the achievement of incentive compatibility: the external auditor is truthful, but more expensive, the internal auditor requires a payment of P^m to refuse the manager's bribe, and they both impose an extra burden on the individual rationality constraint of the manager in the low state of productivity (the manager could be punished by mistake).

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21 Contrast this with Tirole's [1986] model where there is a chance of the auditor (supervisor) getting no signal at all. In that case, a second signal would be valuable.

A new parameter which comes into play is the maximum punishment for the internal auditor, P^{i*}, this bound has the same limited-liability interpretation of P^{m*}.[22]

Let γ be the probability of sending the internal when output is low, d the probability of sending the external when the internal reported a low productivity signal and φ the probability of sending the external when the internal is not sent. The problem for the principal is now to choose[23] $e_1, e_2, t_1, t_2, w, \gamma$, d and φ to:

$$\text{Max}\; q\,\{\theta_1 + e_1 - t_1 + \gamma\,[(1-r)(P^m - w) - r\,\delta\, z] + (1-\gamma)\,[\varphi((1-r)\,P^m - z)]\} +$$

$$(1-q)\,\{\theta_2 + e_2 - t_2\}$$

subject to:

(MIR1) $$t_1 \geq \frac{e_1^2}{2} + P^m\,(1-r)\,[\gamma + \varphi(1-\gamma)]$$

(MIR2) $$t_2 \geq \frac{e_2^2}{2}$$

(MIC) $$t_2 - \frac{e_2^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2} - r\,P^m\,[\gamma + \varphi(1-\gamma)]$$

(CIC) $$w - P^m \geq -\delta\,(P^m + P^i)$$

Definitions:

Let α and β be the same as in proposition 4.

Let

22 Since fraud is punished more harshly by the courts than breach of contract one could argue that $P^{i*} > P^{m*}$. This will increase the value of the external auditor for the principal. (See below.)

23 The principle of maximum deterrence holds for P^i.

$$\eta = \frac{r}{2r-1} \quad \frac{q}{1-q} z - P^i$$

be the maximum value of P^m as a function of r, P^i and z such that if $r \leq \frac{1}{2-q}$ it is optimal not to use any auditor.

Let

$$\rho = \frac{r}{1-r} z - P^i$$

be the maximum value of P^m as a function of r, P^i and z such that if $r \geq \frac{1}{2-q}$ it is optimal to use the internal alone.

Proposition 5: The optimal contract can be divided in four regions according to the values of the informativeness of the auditors' signal r, the maximum punishment for the manager (P^m), the maximum punishment for the internal auditor (P^i) and the cost of the external auditor (z). Two of these regions have three subregions each.

[1] (B0) If $r < \min\{ \frac{1}{2-q} ; \eta^{-1}(P^m,P^i,z) \}$ no auditor is used. See Prop. 1.

[2] If $\eta^{-1}(P^m,P^i,z) \leq r \leq \rho^{-1}(P^m,P^i,z)$ the two auditors are used, the internal with probability 1, the external with probability $d < 1$. The incentive schemes applied in this region are analogous to the ones of Prop. 3 where the cost of the auditor would be dz instead of z.

[3] If $r \geq \max\{ \frac{1}{2-q} ; \rho^{-1}(P^m,P^i,z) \}$ only the internal is used. See Prop. 4.

[4] (FB) If $r = 1$ and $\beta(r) < P^m$ the first best is achieved.

Proof: See appendix.

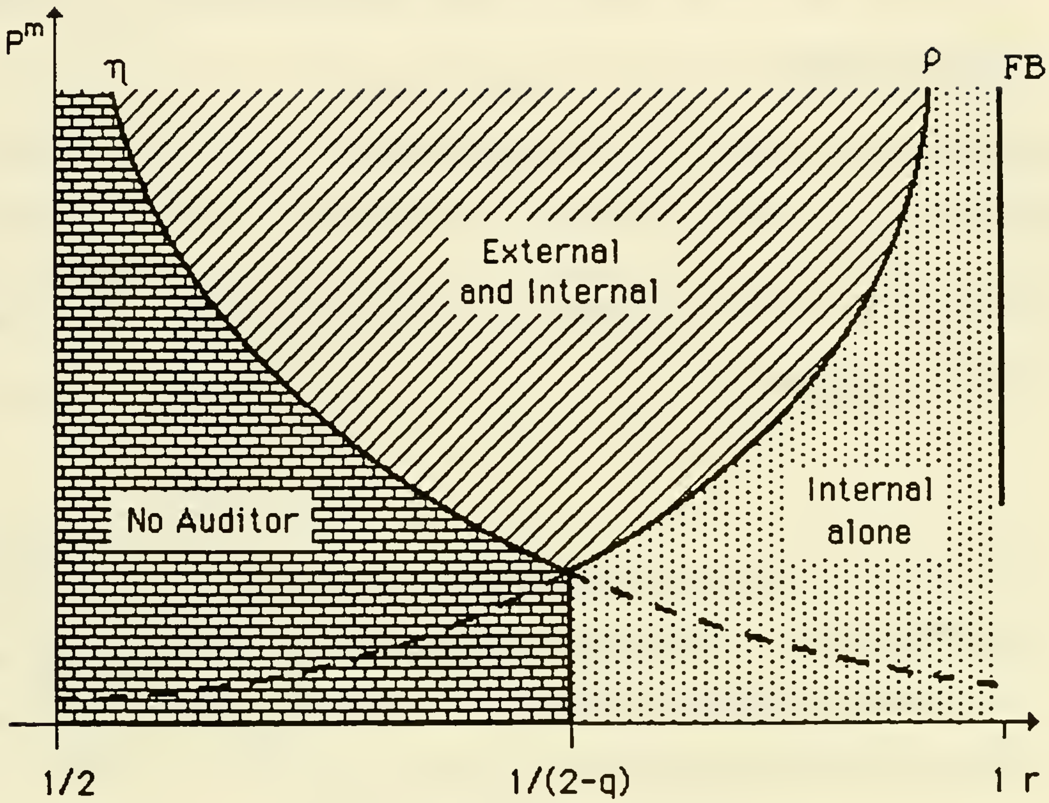

Figure 11. Regions in the Optimal Contract with Two Auditors

Notice that

$$\frac{\partial \eta}{\partial z} > 0 \quad ; \quad \frac{\partial \eta}{\partial P^i} < 0 \quad ; \quad \frac{\partial \rho}{\partial z} > 0 \quad ; \quad \frac{\partial \rho}{\partial P^i}$$

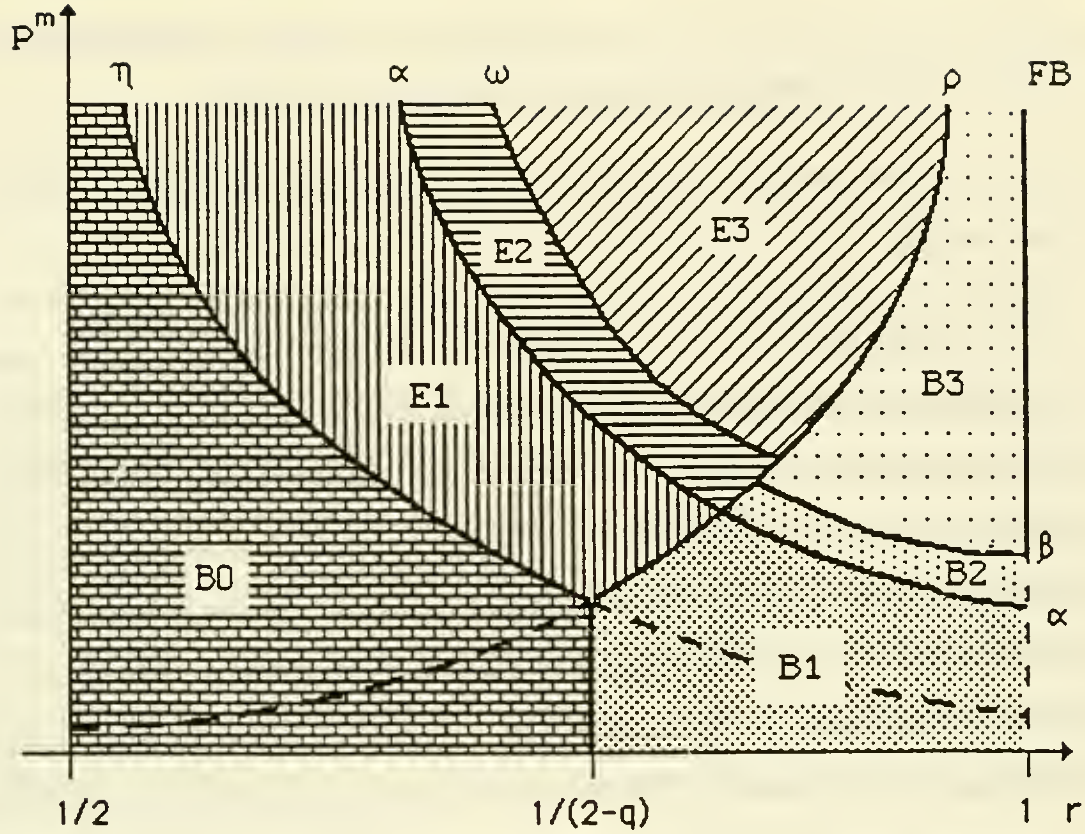

Figure 12. Regions and Subregions in the Optimal Contract with Two Auditors

We find that it is optimal not to use the auditors when their signal is too noisy, but notice that "too noisy" has a different meaning now than in the case without external and costly internal. If auditing is to be used, the principal must decide whether to use the internal alone or both the internal and the external. In the previous model (proposition 4) we compared r with $1/(2\text{-}q)$. This time, however, we have an extra condition due to the possible presence of an external auditor. This is reflected by the comparison of r with $\eta^{-1}(P^m,P^i,z)$. In other words, the principal compares the accuracy of the auditor's signal with a function that is increasing in the maximum punishment to the manager, the maximum punishment to the internal and decreasing in the wage of the external. Some intuition on $\eta^{-1}(\cdot)$ can be obtained by looking at two extreme cases: $z = 0$ (the external is free) and $P^i = \infty$. In these situations, auditing is always useful (recall $r > 1/2$). The case where $z = 0$ is obvious. When P^i is

unbounded, the principal may use the costly external with a very small probability and still induce truthful revelation from the manager and the internal auditor.

If the principal uses the internal alone, the incentive scheme is identical to the one of proposition 4.

If the principal uses the two auditors we are in a situation similar to the one where the internal auditor is truthful but costs dz (the probability of sending the external auditor times his reservation wage). Indeed, the internal will report truthfully, even though he is not paid anything. The reason is that the principal will set the probability of sending the external (d) sufficiently high to discourage him from colluding with the manager when threatened with punishment P^{i*}. The optimal incentive scheme has the three familiar subregions (E1, E2 and E3). The intuition is identical to the one of proposition 3. We also show (lemma 3 in the appendix) that it is never optimal to send the external alone. If he is used at all, he will be used as a "second opinion" required with probability less than 1 [24]. In other words, if the external auditor is used it must be on a random basis. However, we notice that this result depends crucially on the fact that the internal is free.

On the Optimality of Allowing Collusion

So far, we have shown that the principal finds it in his interest to prevent collusion between the auditor and the manager whenever the auditor is hired. In the optimal contract, nobody colludes.

To explore the more plausible case where some deviant behavior persists in equilibrium[25], assume that self-enforcing collusive arrangements are more easily achieved in some states

24 δ is always less than 1 (whenever $P^i > 0$).

25 Without changing the model one could claim that the Revelation Principle is just a solution technique that facilitates the characterization of the optimal contract but does

of the world that in others. For example, take the case where there are two types of auditors: collusive and non-collusive. The collusive auditors will accept bribes with positive expected value and the non-collusive will never take bribes. Then, introducing a binary random variable we could say that with probability c the state of the world supporting collusion obtains and with probability (1-c) side-contracts are not feasible[26].

Suppose for example that with probability c the auditor has pre-contractual ties with the manager of the firm —ties that make side-contracts between them self-enforcing— and that these ties are "per se" illegal. Then, the court will not enforce any contract that is contingent of a report of this variable. Baiman-Evans-Nagarajan say that when contracts based on a "type" report of the auditor are unenforceable, the owner cannot write a contract which will induce the auditor to reveal his type; i.e., the owner cannot write screening contracts. Although we concur with them on the point that a screening contract is not feasible we don't agree with their argument[27].

Even though a report of type may be non-contractible upon, the court could enforce any clause contingent on the outcome of a publicly observed variable which the principal and the auditor can agree to determine. For example, the principal and the auditor may agree (although nobody may know this) that when the auditor wears a red tie he is of the colluding type and when the auditor wears a blue tie he is a non colluder. As long as the tie

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not claim the uniqueness of the characterized optimal contract. In fact, the described mechanism may be a "truth-telling map" of a very complex mechanism where bribes, punishments and rewards happen in equilibrium.

26 Baiman, Evans and Nagarajan claim that since this approach implies that whether or not an individual will collude is not an inherent characteristic of that individual, collusion cannot be avoided by using a screening contract. We will state our disagreement with this approach later.

27 We thank Eric Maskin for suggesting this line of thought.

worn by the auditor is observable by third parties, the court will enforce a contract contingent on its color. So what is it that fails if not the contractibility on type?

What fails in this setup to separate types is the single-crossing, screening or Spence-Mirrlees condition. There is no feature of the auditors which enables the principal to discriminate between them by means of providing different incentives. If he simply asked for type reports promising a high reward for the colluder, every auditor would claim to be a colluder; if he promised a punishment for the colluder, every auditor would claim to be a non colluder. The problem of the two types of auditor as modeled here is that they have the same utility function although different strategy spaces, and different strategy spaces are not enough to screen by means of a menu of contractual arrangements.

There are two models that are interesting to study depending on whether an honest external auditor is available to check the report of the internal auditor. (In this paper we will only explore the first, leaving the second as a topic for further research.) (1) In the first model the principal decides to hire an auditor who is honest with probability (1-c) and who would take bribes with probability c. In order to illustrate our model, we will label this auditor the internal. Internal auditors are usually employees of the firm hired and fired by the manager. Therefore, there might be likely to suffer pressures from management to window-dress the statements of the firm. In our model, with probability (1-c) an internal auditor never takes a bribe and always reports honestly. (2) The second model introduces a second auditor (the police of the police). That auditor is assumed to be always honest but expensive.

For some parameter values, it will be the case that in either of these models the optimal contract will not specify a complete avoidance of collusion. The intuition is that if there is a large probability of non-collusion situations, the principal will prefer to let the few outlayer

cases get away with collusion instead of paying every type of auditor the reward that would deter the few colluding ones.

In the model without the external there would be collusion in equilibrium but it would never be discovered. Everybody would know that in some cases the manager would be able to bribe the auditor but, since the principal has no way of verifying the audit, he cannot stop the bribing. In the model with an honest external, this is not the case. Indeed, the external will be sent with enough probability to deter the "soft" types of auditor (the ones without previous ties to management) but not often enough to deter the "tough" types. Thus, collusion will again occur. The difference is that now sometimes it will be discovered: the external auditor will find that the signal reported by the internal auditor is not correct. Then there will be not only collusion in equilibrium, but collusion that is discovered (and punished) observed in the real world.

When the principal deters collusion nobody shirks or colludes and when an auditor receives a signal implying that the manager shirked, everybody knows it was mistaken. Still, punishment has to be applied to the manager or the contract would not be incentive compatible. This suggests that not everything is bad for the principal when collusion is allowed. Since punishments are only applied by mistake it may be actually good for the principal to allow collusion because it will prevent the application of these punishments. In this section we will explore the soundness of this intuition and show that it must be compared with the loss of control that collusion brings about. Another claim we will examine is that collusive agents might produce a higher payoff for the principal than non-collusive agents.

Can Collusion Enhance Welfare?

Before we study the convenience or not of allowing collusion we will examine the possibility that collusion is welfare-enhancing. We will base our considerations on Robert Klitgaard's insightful book *Controlling Corruption*.

Klitgaard notes that many scholars have argued that corruption can play a useful role. This statement goes beyond saying that the optimal level of corruption is not zero. It is not just that fighting corruption may be so costly as not to be worthwhile, but that corruption itself may create economic, political or managerial benefits. He distinguishes three categories of arguments that *may* make corruption socially beneficial: the economist's reminder, the political scientist's reminder and the manager's reminder. We will describe the first and third since they are the relevant ones for our present concerns.

The economist's reminder. Corrupt payments introduce a mechanism similar to the market system. When goods and services are allocated by queue, politics, random selection, or "merit," corruption may instead allocate goods according to willingness and ability to pay. Corruption may thereby put goods and services in the hands of people who value them the most and who can use them most efficiently. In some sense, then, after the corrupt act those goods and services are more "efficiently" allocated in the economics sense.

The manager's reminder. Corruption may have uses within an organization. If bureaucratic rules are constraining, the organization may sometimes benefit by the employees' corrupt circumvention of the rules.

These reminders, claims Klitgaard, have some common features. First, they refer to the benefits from specific corrupt acts, not from systematic corruption pervading many or most decisions. Second, the putative benefits depend upon the assumption that the corruption transgresses a wrong or inefficient economic policy or gets around imperfections in

organizational rules. In short, *if the prevailing system is bad, then corruption may be good.*

Klitgaard quotes political scientist Samuel Huntington: "In terms of economic growth, the only thing worse than a society with a rigid, overcentralized, dishonest bureaucracy is one with a rigid, overcentralized, honest bureaucracy."

The upshot of this discussion is that corruption can only be helpful as a second-best mechanism when there are some inefficiencies impossible to remove. In the realm of optimal contract theory, where the principal designs mechanisms only subject to information asymmetries collusion will never enhance welfare.

Results

The important decision the principal faces whenever he decides to hire an auditor at all is if he is better off allowing or deterring collusion between the manager and the auditor. If he decides to deter collusion, he will have to pay the auditor a reward at least as large as the punishment imposed on the manager. If he allows collusion he will not pay any reward, but a collusive auditor will never report that the manager should be punished. If the collusive auditor never reports a non-compliance why would the principal find his services useful?

The auditor is useful for the principal on two accounts (1) Not all auditors are collusive and (2) depending on the bargaining power of auditor and manager (we assume the Nash Bargaining Solution) the manager will still suffer a loss when the auditor receives a high-productivity signal after low-productivity production obtained. Namely, the manager will have to pay the auditor a bribe equal to half the punishment; i.e., $P^m/2$. The principal does not get this money, but the deterrence effect of the punishment will still operate on the manager. On the other hand, recall that the principal must compensate the manager for the

expected mistaken punishment (the agent always complies so punishments occur only by mistake.) So the principal may be better off by letting the manager and a collusive auditor play cooperatively.

Our result is that the lower the proportion of colluders, the higher the punishment and the worse the auditor's information, allowing collusion becomes more profitable for the principal. When the probability of having a colluding auditor is low the principal will be reluctant to promise a high reward to the auditor in order to prevent collusion. This is so because with high probability collusion will not happen anyway since the auditor is honest. When the punishment is high our assumption about the Nash bargaining solution becomes very important. Even the collusive auditor will inflict a loss of $P^m/2$ to the manager. Therefore, if the punishment becomes high enough, half of it can be enough to achieve the most profitable second-best contract given the informativeness of the auditor's signal (recall the non-maximum expected deterrence result in proposition 4).

To see why a low correlation between the auditor's signal and the true state of the world makes allowing collusion more profitable for the principal recall that when the manager is low-type and the signal is high the manager will be punished by mistake. In order to get the manager to sign the contract the principal needs to compensate him in advance for these expected mistakes. If the auditor never colluded, since everybody is risk neutral, the principal would simply get his expected payment back. If the auditor always colludes the principal can only achieve coalition incentive compatibility by rewarding the auditor for an amount equal to the punishment imposed on the manager, therefore loosing all his advanced payment to the manager. But in this case, with some probability, the auditor colludes taking only half of the punishment and this is common knowledge. So, the principal has to make a lower advanced payment to the manager. The counter-face of this beneficial effect is that the punishment threat becomes less effective to support the screening of types.

To solve this problem, we will derive a modification of the scheme in proposition 2 but considering now a free unfaithful auditor *and assuming that the principal does not offer any reward and, thus, does not deter collusion* We will compare the profits of this modified scheme with the profits of proposition 4 to study the optimality of allowing collusion.

No Collusion Deterrence by Assumption

We will now allow for a probability $c<1$ that the auditor is unfaithful, that the principal does not offer any reward to the auditor and, thus, does not deter collusion, and the Nash Bargaining solution for the coalition between the unfaithful auditor and the manager. The problem for the principal is now to choose e_1, e_2, t_1, t_2, γ to

$$\text{Max}\ q\,\{\theta_1 + e_1 - t_1 + \gamma\,(1 - c)\,(1 - r)\,P^m\} + (1 - q)\,\{\theta_2 + e_2 - t_2\}$$

subject to:

$$\text{(MIR1)} \qquad t_1 \geq \frac{e_1^2}{2} + \gamma\,(1 - r)\,P^m\,\frac{(2-c)}{2}$$

$$\text{(MIR2)} \qquad t_2 \geq \frac{e_1^2}{2}$$

$$\text{(MIC)} \qquad t_2 - \frac{e_1^2}{2} \geq t_1 - \frac{(e_1 - \Delta\theta)^2}{2} - \gamma\,r\,P^m\,\frac{(2-c)}{2}$$

Notice that this is the same problem as in proposition 2 replacing P^m by $P^m\frac{(2-c)}{2}$. The solution to this problem is given by

Define:

$$\xi(c) = \frac{2 - c - 2q\,(1 - c)}{(2 - q)\,(2 - c) - 2q\,(1 - c)}$$

as the minimum value of r such that it is profitable to use an auditor when the probability of him being a colluder is c.

Notice that $\xi_c > 0$ and that when $c = 0$, $\xi = 1/2$; when $c = 1$, $\xi = 1/(2-q)$.

$$\alpha(A) = \frac{2\,\Delta\theta}{(2-c)(2r-1)}\left[1 - \frac{\Delta\theta}{2q}(2-q)\right]$$

as the value of P^m at which MIR2 becomes binding.

$$\beta(A) = \frac{2\,\Delta\theta}{(2-c)(2r-1)}\left[1 - \frac{\Delta\theta}{2} - \frac{\Delta\theta\,(1-r)}{(2r-1)(2-c)}\right]$$

as the value of P^m at which the benefit of adjusting effort is equal to the cost of increasing the expected punishment (γP^m).

Lemma 6.1: The optimal contract when the principal never deters collusion can be divided in four regions according to the values of r, P^m and c.

[B0] If $r < \xi(c)$, the auditor is not used ($\gamma = 0$) and scheme B0 is applied.

If $1 > r > \xi(c)$, the auditor is used ($\gamma > 0$).

[B1(A)] If $P^m \leq \alpha(A)$, $\gamma = 1$, (MIR2) is slack and scheme B1(A) is applied.

[B2(A)] If $\alpha(A) < P^m \leq \beta(A)$, $\gamma = 1$, (MIR2) binds and scheme B2(A) is applied.

[B3(A)] If $P^m > \beta(A)$, scheme B3(A) is applied.

[FB] If $r = 1$ and $P^m > \beta(A)$ then the first best is achieved.

Definition of Schemes:

Scheme B1(A) is identical to scheme B1.

Scheme B2(A) is identical to scheme B2, except for e_1;

$$e_1^{B2(A)} = \frac{\Delta\theta}{2} + \frac{P\,(2 - c)\,(2r - 1)}{2\,\Delta\theta}$$

Scheme B3(A)

$$\gamma\,(r, P^m) = \frac{\beta(A)}{P^m} \qquad e_1^{B3(A)} = 1 - \frac{\Delta\theta\,(1 - r)}{2r - 1}$$

Proof: See appendix

Remember that we assume here that since the principal does not deter collusion, he only has two options: he can either not use the auditor's report and give a second best incentive scheme (B0) or ask for the auditor's report knowing that this report may be biased. This latter contract may be optimal because the agent who slacks and gets caught by the auditor has to bribe him to avoid the punishment. The amount of the bribe might deter the agent from slacking.

The line ξ determines which incentive schemes the principal chooses: if $r < \xi$, the principal does not use the auditor's report (B0 scheme). If $r \geq \xi$, the auditor's report is used and collusion will occur. Notice that ξ depends on q and c. It depends on q because collusion will only occur when the agent is of type 1 (this happens with probability q) and the (mistaken) signal received by the auditor is the one of type 2. It also depends on c. If $c = 0$, i.e., all auditors are faithful and never collude, $\xi = 1/2$. This means that the principal always uses the auditor report. If $c = 1$, i.e., all auditors accept bribes,

$$\xi = \frac{1}{2-q}.$$

Notice that in this case the region where the auditor's report is used is the same as in proposition 4.

In proposition 4, line $1/(2-q)$ was the level of informativeness of the auditor's signal r where it became more profitable for the principal to punish the agent (by mistake!) than to not use the auditor's report. Recall that $1/(2-q)$ is independent of the punishment level. So

if instead of imposing a loss of P^m on the manager (as it happened in proposition 4) the principal imposes a loss of $\frac{P^m}{2}$ (as in lemma 1 when c = 1) the condition for using the auditor remains invariant.

When the $\xi < r \leq 1$ the auditor's report is used and three different incentive schemes are applied depending on the value of r and P^m. For low r and low P^m, the standard scheme B1 is applied and the rent of the high type is extracted. When r and P^m are sufficiently high, all the rent of the high type manager has been extracted and the principal can use scheme B2(A) to restore the effort of the low type. Finally, if r and P^m are even higher, another expected non-maximum deterrence result obtains with scheme B3(A). The effort of the low type is kept fixed below its first best level however large P^m is. The principal does not use all the punishment capacity he has because the burden that P^m imposes on the MIR1 constraint of the agent becomes too high relative to the potential extra profit gained by restoring the effort. The intuition is similar to the one developed in proposition 4. The cost of increasing P^m through the MIR1 constraint outweighs its benefit through the relaxation of the MIC constraint.

<u>Optimal Collusion Deterrence with a Free Auditor of Unknown Type</u>

The contract of lemma 6.1. was derived assuming that the principal would never find optimal to deter collusion. This assumption is not necessarily true. If we abandon it, we can obtain our main proposition.

Define:

$$\underline{\xi} = \frac{2 - c - 2q(1 - c)}{(2 - q)(2 - c) - 2q(1 - c)}$$

as the as the minimum value of r such that it is profitable to use an auditor when the probability of him being a colluder is c.

$$\bar{\xi} \geq \frac{2q(1-c)+c}{q(2-3c)+2c}$$

as the minimum value of r such that it is profitable to deter collusion. (See the appendix for a more precise definition of $\bar{\xi}$.)

$\beta(D) = \beta$ in proposition 4.

Proposition 6: The optimal contract with a free auditor of unknown type can be divided in four regions according to the values of the informativeness of the auditor's signal r, the maximum punishment for the manager P^m and the probability of having a collusive auditor c. Two of these regions have three sub-regions each.

[1] If $r < \underline{\xi}(c)$, no auditor is used and a scheme B0 is applied.

[2] If $\underline{\xi}(c) < r < \bar{\xi}$, collusion is allowed. The incentive schemes applied in this region are the ones defined in Lemma 6.1.

[3] If $r > \bar{\xi}$, collusion is deterred. The incentive schemes applied in this region are the ones of proposition 4.

[4] If $r = 1$ and $P^m \geq \beta(D)$, the first best is achieved.

Proof: See appendix.

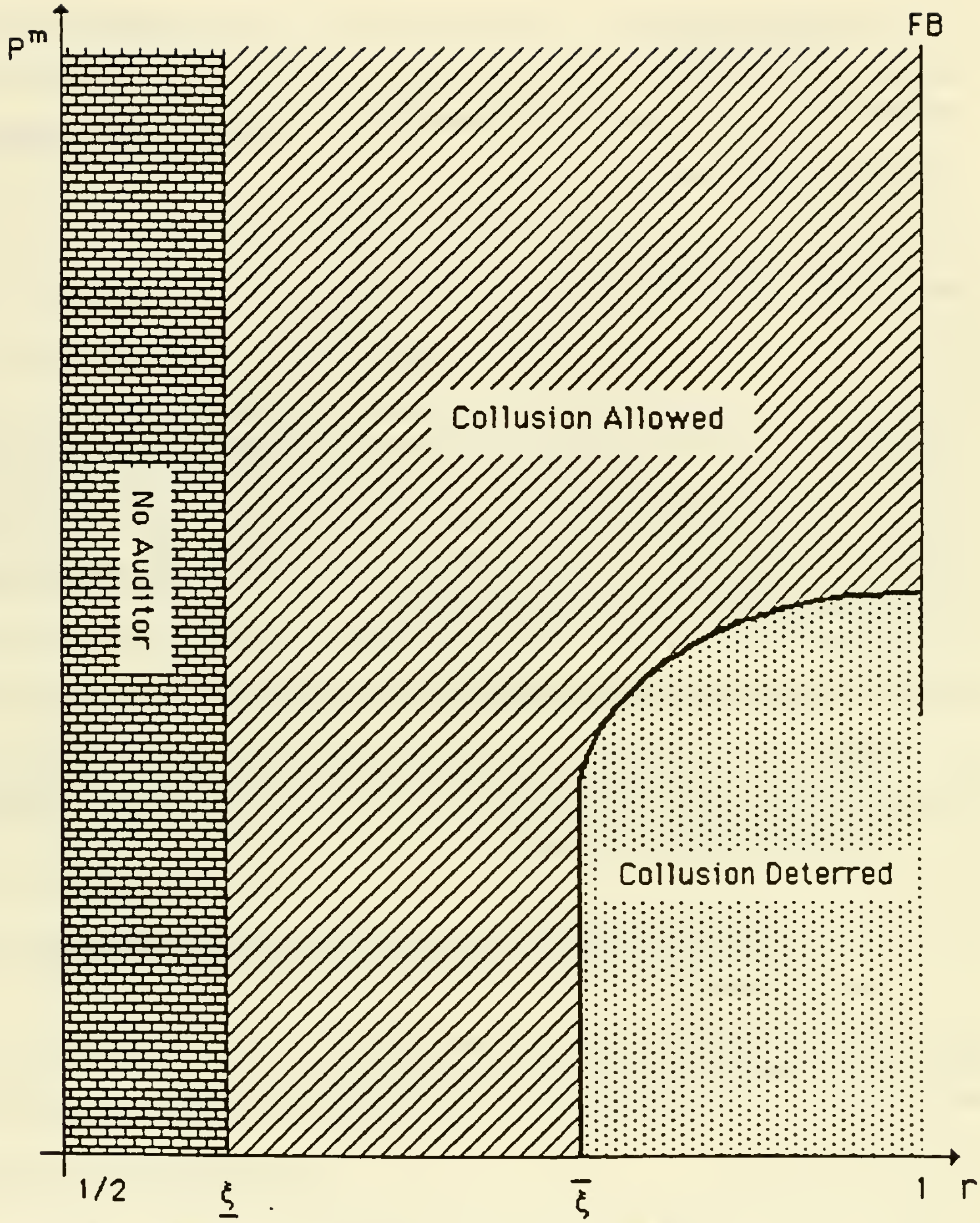

Figure 13. Regions of the Optimal Contract with a Free Auditor of Unknown Type

When the informativeness of the signal is sufficiently bad (i.e., r below $\underline{\xi}$) the principal will find it optimal to not use the auditor. Even though the auditor is free, by submitting the agent to an audit he places him under the risk of being punished by mistake. To satisfy the participation or individual rationality constraint the principal needs to pay the agent the

expected value of this mistaken punishment. This would not influence the contract in the event that the principal got the punishment back from the manager since both are risk neutral. However, if the principal deters collusion the punishment will go to the auditor as in proposition 4 (where we say that if r was less than 1/2-q the auditor would not be used) or if the principal does not deter collusion there will be a transfer to the auditor since the agent will have to pay him $P^m/2$ as a bribe. Hence, when there are many mistakes (r is low) no auditor will be used.

Given the probability of a colluding auditor, the principal will be better off by allowing collusion in the diagonally striped region of the above graph. If the principal decides to deter collusion he will have to pay all the auditors the reward when they report a non-compliance and with probability 1-c this reward was not needed; on the other hand, if he does not deter collusion some auditors will be bribed, reducing the deterrence effect of the punishment. The indifference locus of this tradeoff is reflected in line $\bar{\xi}$.

Depending on the scheme chosen, the subregions —as depicted in the following graph— will be the ones derived in proposition 4 and lemma 6.1:

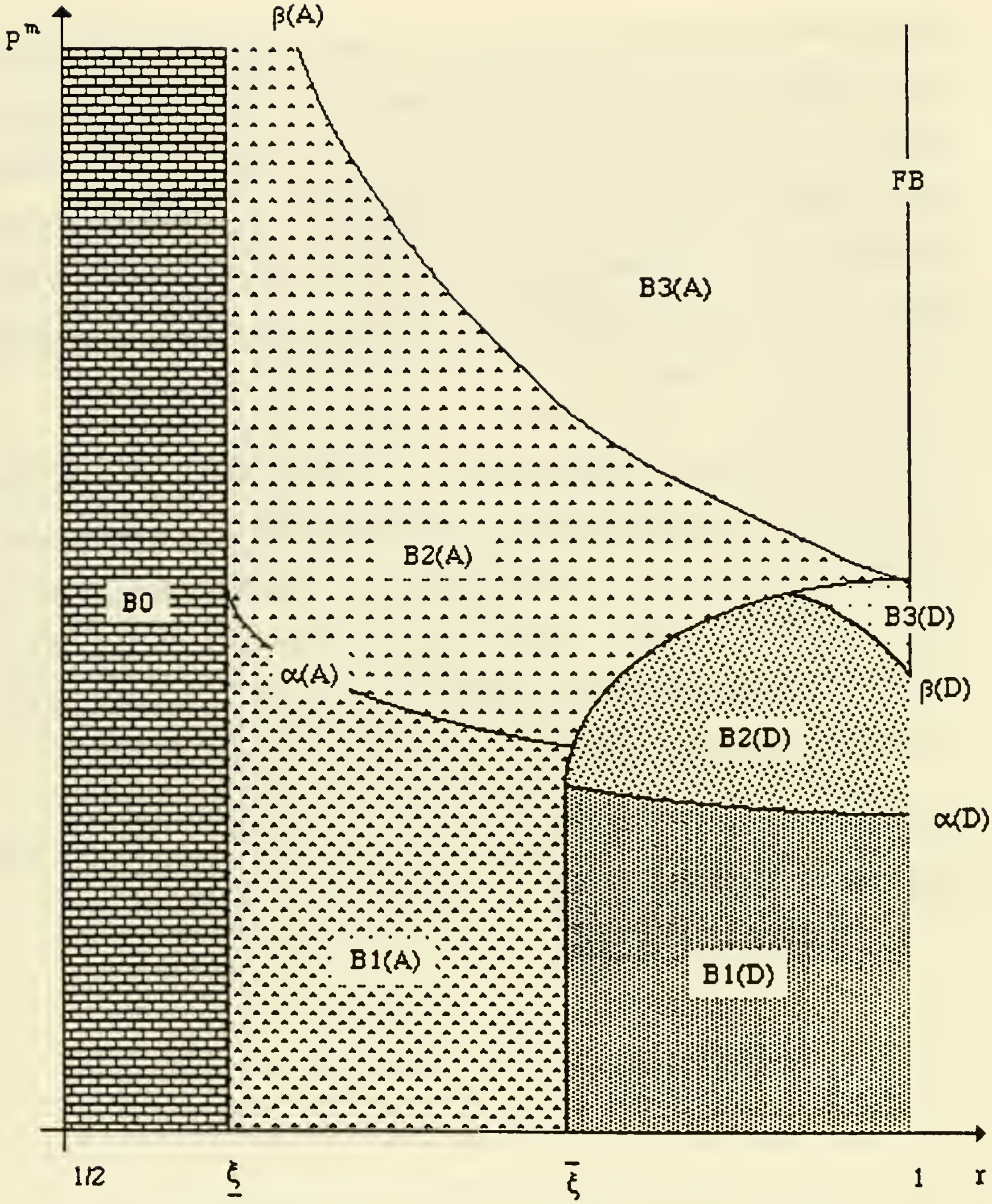

Figure 14. Regions and Subregions of the Optimal Contract with a Free Auditor of Unknown Type

This proposition "merges" the results of proposition 4 and lemma 6.1. The region where no auditor is used is the same as the one derived in lemma 6.1. The definition of $\underline{\xi}$ is identical to the one given for ξ in lemma 6.1. If $\underline{\xi} < r \leq \overline{\xi}$, collusion is allowed and the

scheme of lemma 6.1 is applied. If $r > \overline{\xi}$, the principal finds more profitable to deter collusion applying the scheme of proposition 4.

Two important facts should be noticed:

(1) (Expected) non-maximum deterrence always obtains when the auditor is used. Indeed both schemes don't use maximum deterrence. Therefore, in presence of collusion, the first best is never reached whether the principal deters collusion or not.

(2) If the punishment grows, it always becomes optimal to allow collusion. This result obtains because the effort when collusion is allowed ($e^{B3(A)}$) is higher than the effort when collusion is deterred ($e^{B3(D)}$). The principal can afford to restore the effort more when allowing collusion because the cost of increasing P^m is scaled down by c/2. If the principal deterred collusion he would expect to loose $P^m (1-r) \gamma$ (the expected reward to the auditor). If he allowed collusion, his expected cost would be the difference between the payment required by the low-type manager to sign the contract (see (2-c)/2 in MIR1) and the expected punishment he will collect $(1 - r)(1 - c) \gamma P^m$ when an honest auditor convicts the manager.

Finally, each region has the 3 familiar subregions where rent is extracted, effort is restored and expected punishment is kept constant.

The graphs presented in figures 13 and 14 are an illustration of the optimal contract when $\theta_1 = 1/2, \theta_2 = 1, q = 1/2, c = 1/2$.

The following graphs show how the regions of the optimal contract change with c.

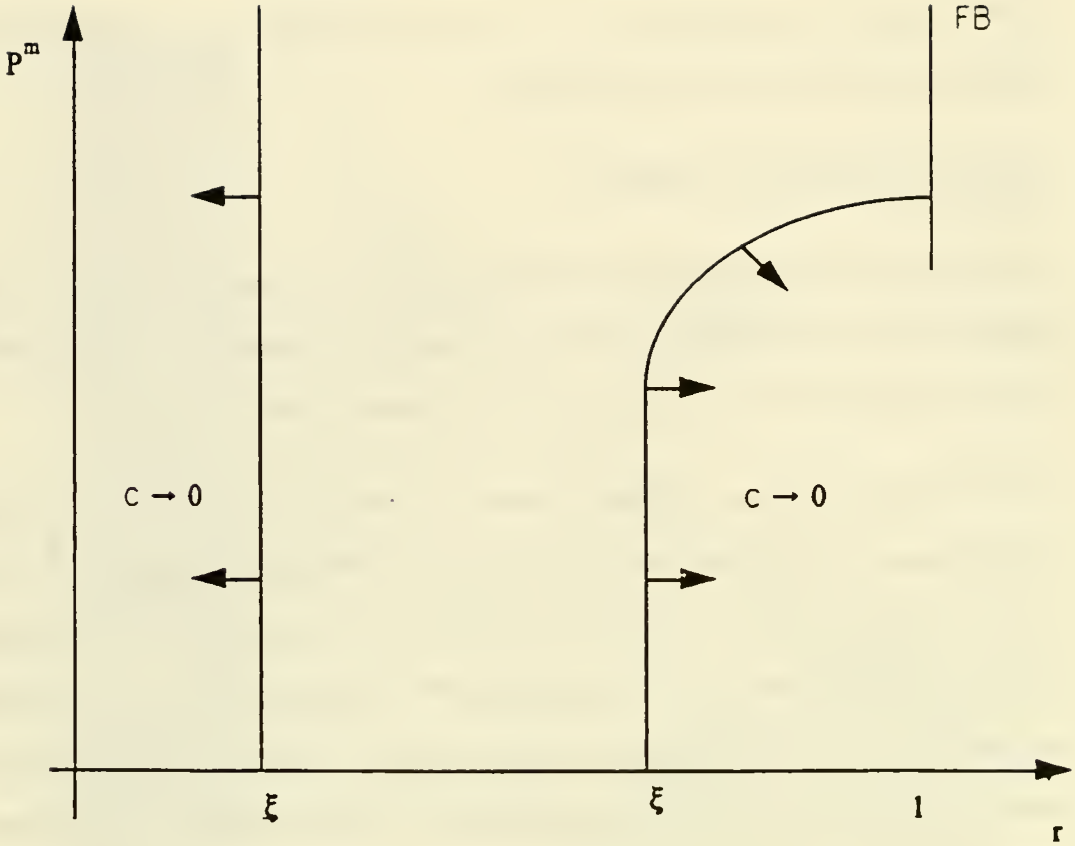

Figure 15. Regions of the Optimal Contract with a Free Auditor of Unknown Type when $c \rightarrow 0$

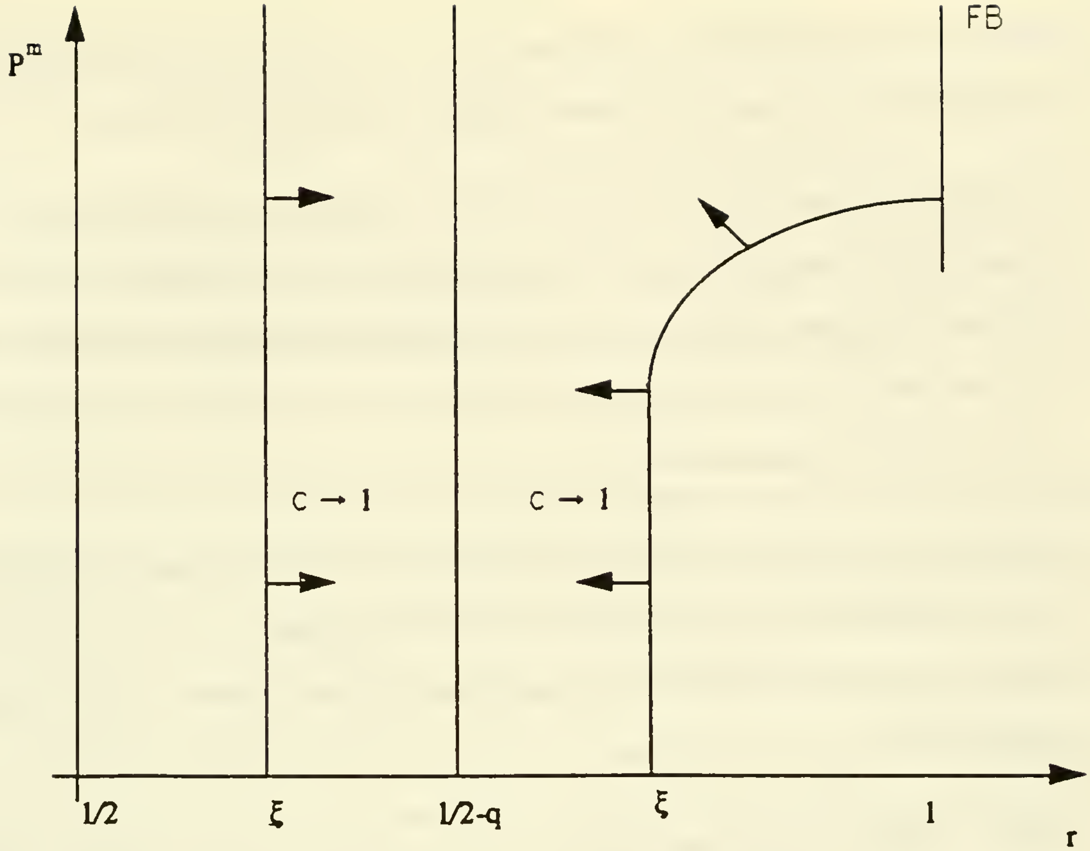

Figure 16. Regions of the Optimal Contract with a Free Auditor of Unknown Type when $c \rightarrow 1$

When the proportion of collusive auditors goes down ($c \rightarrow 0$), the contract resembles the standard second best contract with a faithful auditor.(see proposition 2). At the limit, if $c=0$, no collusion ever occurs and the optimal contract always uses the free honest auditor (see figure 15). At the other limit, if $c=1$, the principal always has to worry about collusion and the optimal contract is identical to the one of proposition 4 (see figure 16).

Conclusions and Directions for Further Research

We have analyzed the role of auditing in a simplified hierarchy (principal-auditor-manager) in which the auditor and the manager can collude; we have distinguished between internal

(costless, self-interested) and external (costly, truthful) auditors and developed an audit technology which is imperfect and allows forging of evidence under collusion.

We have shown that the internal auditor alone may be useful to the principal even if he can collude with the manager. The possibility of collusion imposes an additional cost on the use of the auditor but under certain parameter configurations the principal prefers to bear it. We have also found that the external auditor is always hired on a random basis because his role is mainly to police the internal auditor.

We have also shown that expected maximum deterrence is not necessarily optimal in the presence of collusion. The intuition is that increasing the punishment for the manager raises his effort but also makes collusion more profitable: the bribe tends to be higher and thus harder to deter.

Finally, we have presented a model where the principal has to decide whether to allow or deter collusion. Deterring collusion is costly for the principal because he has to reward his auditor for not accepting a potential bribe from the manager. If the principal believes that his auditor is honest with a sufficiently high probability, he will not try to deter collusion and therefore let the collusive auditor and the manager collude. Even in such a case, hiring an auditor is useful because the manager knows that shirking may cost him the punishment if the auditor is honest and the bribe if the auditor is collusive and gets a signal of shirking.

In this analysis we have assumed that the bargaining power of the manager and the auditor was identical and, thus, the Nash bargaining solution obtained within the coalition formed by these two agents. More generally, we could model the bargaining power of the manager and the auditor by a variable $p \in [0,1]$ and a transfer (bribe) of $W+(P^m-W)p$. In this setup, $p = 0$ means that the manager has all the bargaining power and can bribe the auditor by just matching the principal's offered reward, $p = 1$ means that the auditor has all the

bargaining power and will extract a payment from the manager equal to the potential punishment that the manager would suffer if the auditor reported his signal to the principal.

We have also assumed that the principal only offers a reward when his purpose is to deter collusion. This need not be optimal since the reward offered by the principal will alter the bargaining process within the coalition. In a future version we plan to investigate this issue.

One objection to the contracts characterized in our paper is that the resulting equilibrium strategies are not renegotiation-proof. Once a low output has been revealed, the principal and the manager may prefer to renegotiate and reach a Pareto-superior allocation (e.g., the principal does not audit and the manager gives the principal one half of the expected penalty). The original contract would be renegotiated and this would alter the manager's incentive to tell the truth in the first place. To avoid the renegotiation issue, we have given the principal commitment power over the probability of audit. Although we plan to characterize the optimal renegotiation-proof contract in a future extension of this paper, for the moment we will accept this simplifying assumption.

An alternative to the ad-hoc power of commitment with which we endow the principal is to invoke a reputation argument: in an infinite supergame where the principal and the manager could only sign one-period contracts the original contract would be renegotiation-proof. Introducing reputation to make endogenous assumptions about the principal and the auditors opens the issue of dynamics. We are exploring the consequences of considering a two-period model in which the "faithfulness" of the external and the "self-interest" of the external are a consequence of the utility-maximizing behavior.

Another alternative is to solve the renegotiation-proof contract relying on mixed strategies for the two types of manager. It is a well known result of the crime-deterrence literature that a game where a principal who cannot commit to a (possibly contingent) investigation

strategy by a costly enforcer polices an agent who may or may not comply with the provisions of a contract or law does not have a pure-strategy equilibrium. If the agent complies with probability one, the enforcer should not be used. If the enforcer is not used, the agent should not comply. If the enforcer is used with probability one, the agent should comply. If the agent complies the enforcer should not be used.

Our model can be extended in a variety of directions. If we consider a costly self-interested internal auditor, for example, the probability of sending the external alone need not be zero any more. If all auditors behave strategically the interesting question arises as to whether it is possible to police the police without falling in an infinite regress.

Another interesting direction would be to allow the auditors' information to be imperfectly correlated. A new trade-off would appear: the internal could be punished by mistake and his individual rationality constraint would gain importance (he won't work unless he is compensated for the possible mistakes of the external).

There is an important reason to study this extension. In our model, we assume that the external auditor never colludes. But even if he could collude, the optimal contract would not be altered since the external auditor has exactly the same information as the internal auditor. To prevent the external from colluding we need only to add a new coalition incentive compatibility constraint promising a reward for truthful revelation. As in the model of Demski and Sappington, whatever that reward is the principal never has to pay it because the external can only confirm the internal's report [28].

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28 The implicit assumption here is that any side contract that the external could sign with the agent or the internal is non-enforceable. Denying him this commitment capacity is crucial since if he could bind himself to take, say, half of the bribe; the coalition would be in a Pareto superior situation.

If the information of the two auditors is imperfectly correlated, the study of a self-interested external becomes more interesting (and more complex). In this case several other types of coalitions between the manager and the external, the external and the internal and the three of them must be considered.

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Appendix

Proof of proposition 3

For proving proposition 3 we will need the following lemmas:

Lemma 3.1: In region B1, $\gamma = 1$.

Proof: In region B1, the slope of the profit function with respect to P^m when more rent is extracted from the high type (with $\gamma = 1$) is

$$\frac{d\pi^1}{dP^m} = \frac{\partial\pi^1}{\partial t_2^{B1}} \frac{\partial t_2^{B1}}{\partial P^m} = (1 - q)\ (2r - 1)$$

The slope of the profit function with respect to P^m when γ is reduced (and no rent is extracted) is [29]

$$\frac{d\pi^1}{dP^m} = \frac{\partial\pi^1}{\partial\gamma} \frac{\partial\gamma}{\partial P^m} = q\ \frac{z}{P^m}$$

Decreasing γ below 1 would only be profitable if

$$q\ \frac{z}{P^m} \geq (1 - q)\ (2r - 1) \Leftrightarrow z \geq \frac{1 - q}{q}\ (2r - 1)\ P^m$$

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29 To achieve incentive compatibility, $\gamma\ P^m$ must equal a constant k,

$k = \dfrac{t_1^{B0} - g(e_1^{B0})}{r}$. Therefore, $\gamma = \dfrac{k}{P^m}$ and $\dfrac{\partial\gamma}{\partial P^m} = -\dfrac{k}{(P^m)^2}$

But at the limit where $\gamma = 1$, $k = P^m$ and then

$\dfrac{\partial\gamma}{\partial P^m} = -\dfrac{1}{P^m}$

But this condition is true when it is profitable not to use the auditor at all. (See definition of κ below.) n

Lemma 3.2: If the cost of the auditor is sufficiently low for the principal to use him at all, there exists a punishment P^m such that the probability γ of sending the auditor is less than one and this occurs before the first best level of effort is reached. Formally,

For $z \leq \frac{1-q}{q}(2r-1)P^m$, there exists $P^m < P^m|_{FB}$ such that $\gamma < 1$, where $P^m|_{FB}$ is the minimum punishment for which $e_1 = 1$ in the optimal contract.

Proof: In lemma 1 we have shown that this can only happen in region B2.

In B2, the slope of the profit function with respect to P^m when effort is adjusted is positive for punishments below the level necessary to achieve the first best and smoothly becomes zero at the first best. (The profit function is increasing and concave in P^m)

On the other hand, the slope of the profit function with respect to P^m when the probability of sending the auditor is adjusted is $(q\,z)/P^m$, which is positive fo all P^m. n

Proof of Proposition 3 (continued)

The Lagrangian for this problem is:

$$L = q\{\theta_1 + e_1 - t_1 + \gamma[(1-r)P^m - z]\} + (1-q)\{\theta_2 + e_2 - t_2\} + \lambda_1\ \{t_1 - \frac{e_1^2}{2} - \gamma(1-r)P^m\}$$

$$+ \lambda_2\{t_2 - \frac{e_2^2}{2}\} + \lambda_3\{t_2 - \frac{e_2^2}{2} - t_1\} + \frac{(e_1 - \Delta\theta)^2}{2} + \gamma\, r\, P^m\} + \lambda_4\{1-\gamma\}$$

with the additional constraints:

$e_1 \geq 0 \quad e_2 \geq 0 \qquad t_1 \geq 0 \qquad t_2 \geq 0 \quad 0 \leq \gamma \leq 1$

$\lambda_1 \geq 0 \quad \lambda_2 \geq 0 \qquad \lambda_3 \geq 0 \qquad \lambda_4 \geq 0$

The Kuhn-Tucker conditions for maximization are:

$$\frac{\partial \mathcal{L}}{\partial e_1} = q - \lambda_1 e_1 + \lambda_3 (e_1 - \Delta\theta) \leq 0 \quad ; \quad e_1 \frac{\partial \mathcal{L}}{\partial e_1} = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = (1 - q) - (\lambda_2 + \lambda_3) e_2 \leq 0 \quad ; \quad e_2 \frac{\partial \mathcal{L}}{\partial e_2} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial t_1} = -q + \lambda_1 - \lambda_3 \leq 0 \quad ; \quad t_1 \frac{\partial \mathcal{L}}{\partial t_1} = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial t_2} = -(1 - q) + \lambda_2 + \lambda_3 \leq 0 \quad ; \quad t_2 \frac{\partial \mathcal{L}}{\partial t_2} = 0 \tag{4}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \gamma} &= q (1 - r) (P^m - w) - \lambda_1 (1 - r) P^m \\ &\quad + \lambda_3 r P^m - \lambda_4 - q z \leq 0 \\ &\quad \gamma \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \end{aligned} \tag{5}$$

Plus the constraints and their complementary slackness conditions.

When the auditor is not used, $\gamma = 0$ and by complementary slackness condition, $\lambda_4 = 0$.

Using eq. (1) - (4), $\lambda_1 = 1$ and $\lambda_3 = 1 - q$.

By eq. (5), $P^m [q (1 - r) - (1 - r) + (1 - q) r] \leq q z$

This implies that $P^m \leq \dfrac{q z}{(2r - 1)(1 - q)} = \kappa(r,z)$

Below κ, the auditor is not used, and the scheme B0 (see proposition 2) is applied.

Line α and schemes B1 and B2 are the same as in proposition 2.

To find ω, we will find e_1 in region E3 and equate it with e_1 in region B2.

In region E3, the auditor is used with probability less than 1 (see lemmas 3.1 and 3.2), so $\lambda_4 = 0$.

By eq. (3) and (5), $\lambda_1 = \dfrac{q\,[P^m (2r - 1) + z]}{P^m (2r - 1)}$

$$\lambda_3 = \lambda_1 - q = \frac{q\,z}{P^m (2r - 1)}$$

By eq. (1) $\qquad e_1 = 1 - \dfrac{z\,\Delta\theta}{P^m (2r - 1)}$

In region B2, $e_1 = \dfrac{P^m (2r - 1)}{\Delta\theta} + \dfrac{\Delta\theta}{2}$ (See proposition 2)

equating the above two expressions we find

$$\omega(r,z) = \frac{\Delta\theta \left[(2 - \Delta\theta) + \sqrt{(\Delta\theta - 2)^2 - 16\,z} \right]}{4\,(2r - 1)}$$

To find γ we use the IR1 and IC constraints:

$$\gamma = \frac{\Delta\theta \left[(2 - \Delta\theta)\, P^m\, (2r - 1) - 2\,z\,\Delta\theta \right]}{2\, P^{m^2}\, (2r - 1)^2}$$

Since at ω, $\gamma = 1$ and in region E3 $\dfrac{\partial\gamma}{\partial P^m} < 0 \Rightarrow \gamma < 1$ in region E3. n

Proof of proposition 4

The Lagrangian for this problem is:

$$L = q\,\{\theta_1 + e_1 - t_1 + \gamma\,(1 - r)\,(P^m - w)\} + (1 - q)\,\{\theta_2 + e_2 - t_2\} +$$

$$\lambda_1\,\{t_1 - \frac{e_1^2}{2} - \gamma\,(1 - r)\,P^m\} + \lambda_2\,\{t_2 - \frac{e_2^2}{2}\} + \lambda_3\,\{t_2 - \frac{e_2^2}{2} - t_1) +$$

$$\frac{(e_1 - \Delta\theta)^2}{2} + \gamma\,r\,P^m\} + \lambda_4\,\{w - P^m\} + \lambda_5\,\{1 - \gamma\}$$

with the additional constraints:

$e_1 \geq 0 \quad e_2 \geq 0 \quad t_1 \geq 0 \quad t_2 \geq 0 \quad w \geq 0$

$0 \leq \gamma \leq 1 \quad \lambda_1 \geq 0 \quad \lambda_2 \geq 0 \quad \lambda_3 \geq 0 \quad \lambda_4 \geq 0 \quad \lambda_5 \geq 0$

given that γ and P^m always appear together, we fix P^m at P^{m*} and use γ as the argument for maximization. The alternative choice —to fix $\gamma = 1$ and let P^m vary— does not alter the results because we assume that the reservation wage of the internal auditor is zero.

The Kuhn-Tucker conditions for maximization are:

$$\frac{\partial \mathcal{L}}{\partial e_1} = q - \lambda_1 e_1 + \lambda_3 (e_1 - \Delta\theta) \leq 0 \quad ; \quad e_1 \frac{\partial \mathcal{L}}{\partial e_1} = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = (1 - q) - (\lambda_2 + \lambda_3) e_2 \leq 0 \quad ; \quad e_2 \frac{\partial \mathcal{L}}{\partial e_2} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial t_1} = -q + \lambda_1 - \lambda_3 \leq 0 \quad ; \quad t_1 \frac{\partial \mathcal{L}}{\partial t_1} = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial t_2} = -(1 - q) + \lambda_2 + \lambda_3 \leq 0 \quad ; \quad t_2 \frac{\partial \mathcal{L}}{\partial t_2} = 0 \qquad (4)$$

$$\frac{\partial \mathcal{L}}{\partial w} = -q (1 - r) \gamma + \lambda_4 \leq 0 \quad ; \quad w \frac{\partial \mathcal{L}}{\partial w} = 0 \qquad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = q (1 - r) (P^m - w) - \lambda_1 (1 - r) P^m + \lambda_3 r P^m$$

$$- \lambda_5 \leq 0 \quad ; \quad \gamma \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \qquad (6)$$

Plus the constraints and their complementary slackness conditions

Suppose $\gamma = 0$, then $\lambda_5 = 0$ by complementary slackness condition and, using (6)

$$q (1 - r) (P^m - w) - \lambda_1 (1 - r) P^m + \lambda_3 r P^m \leq 0 \quad (7)$$

If $\gamma = 0$, the auditor is not used. Therefore the optimal scheme is B0 and we can let $w = P^m$ w.l.o.g.

In B0, $\lambda_1 = 1$ and $\lambda_3 = 1 - q$, equation (7) then becomes

$$r \leq \frac{1}{2 - q}$$

Suppose $\gamma = 1$, then, schemes B1, B2 and the line $\alpha(r)$ are the same as in proposition 2 since the relevant FOC are identical. Also, by the coalition incentive compatibility constraint $P^m = w$.

Suppose $0 < \gamma < 1$ (i.e., we are in region B3). Using equations (1) to (4),

$$\lambda_1 = \lambda_3 + q \qquad (3)$$

$$q - \lambda_1 e_1 + \lambda_3 \ (e_1 - \Delta\theta) = 0 \qquad (1)$$

which imply that $\lambda_3 = \dfrac{q\,(1 - e_1)}{\Delta\theta}$

since $\gamma < 1$, $\lambda_5 = 0$. By the coalition incentive compatibility constraint, $P^m = w$ and using (6) we find

$$- P^m \lambda_1 (1 - r) + P^m r \lambda_3 = 0$$

which implies that

$$e_1 = 1 - \frac{\Delta\theta\ (1 - r)}{2r - 1}$$

Note that e_1 in region B3 is independent of P^m and equal to e_1 in region B2 when

$$P^m = \frac{\Delta\theta\ (4r - \Delta\theta - 2)}{2\ (2r - 1)^2}$$

This defines the line $\beta(r)$, the border between regions B2 and B3.

It is easy to check that $\alpha(r) = \beta(r)$ when $r = \frac{1}{2 - q}$ and that $\alpha(r) < \beta(r)$ when $\frac{1}{2 - q} < r < 1$.

The function $\beta(r)$ is increasing if

$$-4 r^2 + (4 + 2 \Delta\theta) r - (\Delta\theta + 1) > 0$$

and decreasing if

$$-4 r^2 + (4 + 2 \Delta\theta) r - (\Delta\theta + 1) < 0$$

This means that $\beta(r)$ is increasing in the interval

$$\left[\frac{1}{2} , \frac{1 + \Delta\theta}{2} \right]$$

and decreasing when r exceeds the upper bound of the interval.

Therefore, depending on the value of $\Delta\theta$ satisfying the no shut down condition[30], $(1+\Delta\theta)/2$ could be less than $1/(2-q)$, between $1/(2-q)$ and 1, or greater than 1. This gives rise to the following 3 graphs:

30 The maximum $\Delta\theta$ for no shut down is $q/(1-q)$. Therefore the function $\beta(r)$ starts decreasing at most when $r = 1/(2-2q)$.

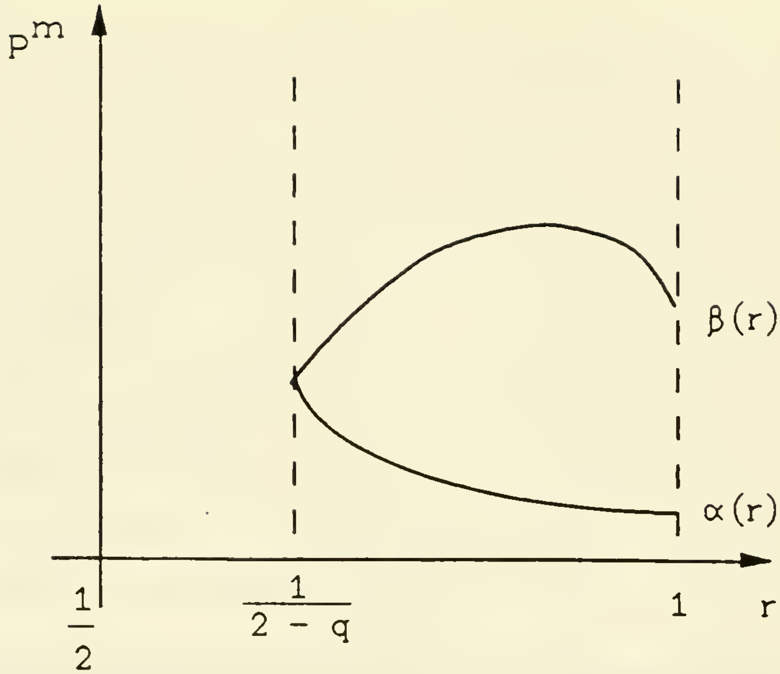

Figure A1.a. First Possible Shape of β(r)

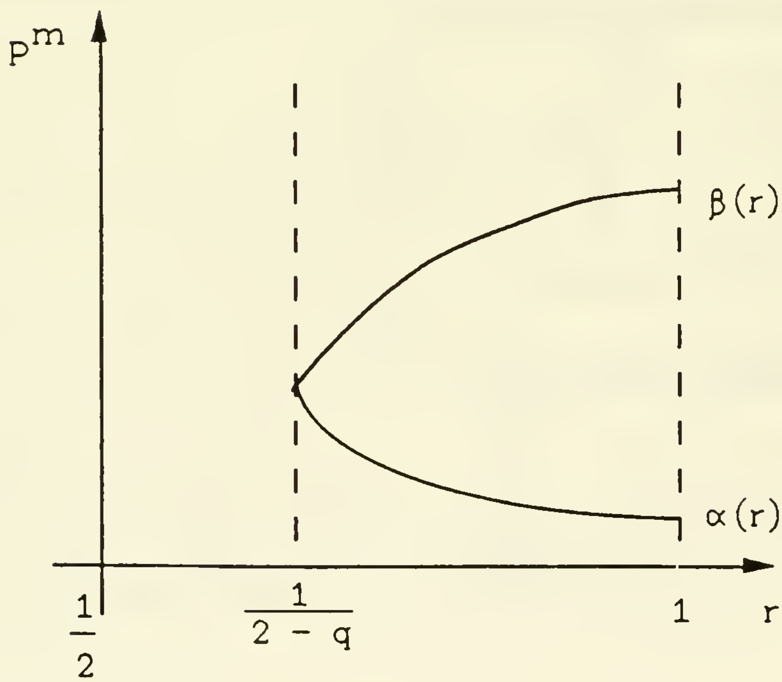

Figure A1.b. Second Possible Shape of β(r)

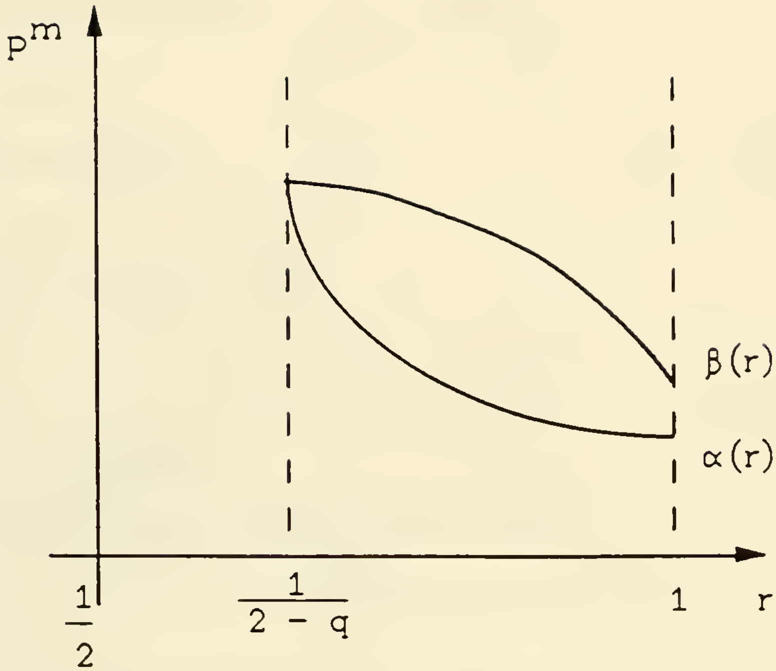

Figure A1.c. Third Possible Shape of $\beta(r)$

Note that such graphs imply a rate of increase in the effort function different in region B2 and B3. Indeed, it is easy to check that

$$\frac{\partial^2 e_1^{B2}}{\partial(r)^2} = 0 \quad \text{and} \quad \frac{\partial^2 e_1^{B3}}{\partial(r)^2} < 0$$

Proof of Proposition 5

The Lagrangian for this problem is:

$$L = q\,\{\theta_1 + e_1 - t_1 + \gamma\,[(1-r)(P^m - w) - r\,\delta\,z] + (1-\gamma)\,[\,\varphi\,((1-r)\,P^m - z)]\} +$$

$$(1-q)\,\{\theta_2 + e_2 - t_2\} + \lambda_1\,\{t_1 - \frac{e_1^2}{2} - (1-r)\,P^m\,[\gamma + \varphi\,(1-\gamma)]\} +$$

$$\lambda_2 \{t_2 - \frac{e_2^2}{2}\} + \lambda_3 \{t_2 - \frac{e_2^2}{2} - t_1 + \frac{(e_1 - \Delta\theta)^2}{2} + r\, P^m [\gamma + \varphi (1 - \gamma)]\} +$$

$$\lambda_4 \{w + \delta P^i - (1 - \delta) P^m\} + \lambda_5 \{1 - \gamma\} + \lambda_6 \{1 - \delta\} + \lambda_7 \{1 - \varphi\}$$

with the additional constraints:

$e_1 \geq 0$	$e_2 \geq 0$	$t_1 \geq 0$	$t_2 \geq 0$
$\gamma \geq 0$	$\delta \geq 0$	$\varphi \geq 0$	$w \geq 0$
$\lambda_1 \geq 0$	$\lambda_2 \geq 0$	$\lambda_3 \geq 0$	$\lambda_4 \geq 0$
$\lambda_5 \geq 0$	$\lambda_6 \geq 0$	$\lambda_7 \geq 0$	

where γ is the probability of sending the internal auditor when output is low, δ the probability of sending the external auditor when the internal auditor reported low productivity and φ the probability of sending the external alone.

The Kuhn-Tucker conditions for maximization are:

$$\frac{\partial \mathcal{L}}{\partial e_1} = q - \lambda_1 e_1 + \lambda_3 (e_1 - \Delta\theta) \leq 0 \quad ; \quad e_1 \frac{\partial \mathcal{L}}{\partial e_1} = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = (1 - q) - (\lambda_2 + \lambda_3) e_2 \leq 0 \quad ; \quad e_2 \frac{\partial \mathcal{L}}{\partial e_2} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial t_1} = -q + \lambda_1 - \lambda_3 \leq 0 \quad ; \quad t_1 \frac{\partial \mathcal{L}}{\partial t_1} = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial t_2} = -(1 - q) + \lambda_2 + \lambda_3 \leq 0 \quad ; \quad t_2 \frac{\partial \mathcal{L}}{\partial t_2} = 0 \qquad (4)$$

$$\frac{\partial \mathcal{L}}{\partial w} = -q\, (1 - r)\, \gamma + \lambda_4 \leq 0 \quad ; \quad w \frac{\partial \mathcal{L}}{\partial w} = 0 \qquad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = q\ \{[(1 - r)\ (P^m\ - w) - r\delta z] - [\varphi((1 - r)\ P^m\ - z)]\}$$

$$- \lambda_1\ (1 - r)\ (1 - \varphi)\ P^m\ + \lambda_3\ P^m\ r(1 - \varphi) - \lambda_5 \leq 0$$

$$\gamma\ \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial \delta} = -q\ r\ z\ \gamma + \lambda_4\ (P^i + P^m\ ) - \lambda_6 \leq 0\ ; \quad \delta\ \frac{\partial \mathcal{L}}{\partial \delta} = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = q\ (1 - \gamma)\ [(1 - r)\ P^m - z] - \lambda_1\ (1 - r)\ (1 - \gamma)\ P^m$$

$$+ \lambda_3\ P^m\ r\ (1 - \gamma) - \lambda_7 \leq 0 \quad ; \quad \varphi\ \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \tag{8}$$

Plus the constraints and their complementary slackness conditions

For proving the proposition we will use the following three lemmas:

Lemma 5.1: If the outsider is sent after the insider, the conditional probability of sending the outsider is $\delta = \dfrac{pm - w}{pm + pi}$.

Proof: If the outsider is sent after the insider, $\delta > 0$. Suppose that $1 > \delta > 0$.

$1 > \delta \Rightarrow \lambda 6 = 0$ by the complementary slackness condition

$0 < \delta \Rightarrow \dfrac{\partial \mathcal{L}}{\partial \delta} = 0$ by the complementary slackness condition

Then, $\lambda 4 = \dfrac{q\,r\,z\,\gamma}{pm + pi} > 0$

$\lambda 4 > 0$ implies that $\delta = \dfrac{pm - w}{pm + pi}$ by the CIC constraint.

Next, we will show that $\delta = 1$ leads to a contradiction.

If $\delta = 1$, $\lambda_4 = 0$ because $w + P^i > 0$ (from CIC constraint),

but then, from (7), $\lambda_6 = -q\,r\,z\,\gamma < 0$. Contradiction since $\lambda_6 \geq 0$. ∎

Lemma 5.2: The reward for the internal auditor (w) can only be 0 or P^m.

Proof: Substituting our above result for δ in the objective function, the principal's problem involves the following maximization:

$$\max_{0 \leq w \leq P^m} \quad (1-r)\,(P^m - w) - r\,\frac{(P^m - w)}{(P^m + P^i)}\,z$$

whose solution can be always found at a corner:

$$\text{If } r < \frac{P^m + P^i}{P^m + P^i + z} \quad \text{then} \quad w = 0$$

$$\text{If } r > \frac{P^m + P^i}{P^m + P^i + z} \quad \text{then} \quad w = P^m$$

$$\text{If } r = \frac{P^m + P^i}{P^m + P^i + z} \quad \text{then} \quad w \in [0, P^m] \;∎$$

Lemma 5.3: The external auditor is never used alone. I.e., $\varphi = 0$.

Proof: We will show that the scheme where the external is used alone is strictly dominated by the scheme where both auditors are used.

Let υ be the minimum probability of an (honest) audit necessary to achieve manager incentive compatibility.

The cost of achieving MIC by using the external alone would be υz.

The cost of achieving MIC by using the two auditors would be $\upsilon\delta z$. Since by lemma 1, $\delta < 1$, $\upsilon\delta z < \upsilon z$. ■

(Notice that this result depends crucially on the fact that the internal is free.)

Proof of proposition 5 (continued):

We must consider 2 alternative ways by which the principal could achieve incentive compatibility: (i) paying the internal auditor a reward no lower than the maximum bribe (P^m) or (ii) sending an external with probability (δ) high enough (with respect to P^m and P^i) to deter collusion.

As we have proved in lemmas 5.1 and 5.2 the optimal δ is $\frac{P^m - w}{P^m + P^i}$ and it is never strictly optimal to use a "mix" of the two methods; i.e., w and δ strictly positive. This implies that w is either 0 or P^m.

(i) If $w = P^m$, then $\delta = 0$. Then, equation (6) implies that

$$p \leq \frac{1}{2 - q}$$

(ii) If $w = 0$, then $\delta = \frac{P^m}{P^m + P^i}$. Then, equation (6) implies that

$$P^m \leq \frac{r}{2r - 1} \frac{q}{1 - q} z - P^i = \eta(r, P^i, z)$$

The condition for not using any auditor is then,

$$r \leq \min \left\{ \frac{1}{2 - q} \; ; \; \frac{1}{2 - \dfrac{q\,z}{(1 - q)\,(P^m + P^i)}} = \eta^{-1}(P^m, P^i, z) \right\}$$

When the principal hires only the internal auditor, $\gamma = 1$ [31], $\delta = 0$ (this implies that $\lambda_6 = 0$) and $w = P^m$.

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31 Recall that when the internal is costless we can adjust P^m setting γ always equal to one without loss of generality.

From equation (5), $\lambda_4 = q\,(1 - r)$

Substituting in (7), $r \geq \dfrac{p^m + p^i}{p^m + p^i + z} = \rho^{-1}(P^m)$

And this takes us back to the schemes B1, B2 and B3 derived in proposition 4.

At this point we can show that η and ρ cross when

$$\eta(r) = \frac{r}{2r-1}\,\frac{q}{1-q}\,z - P^i = \frac{r}{1-r}\,z - P^i = \rho(r)$$

which occurs at $r = \dfrac{1}{2-q}$

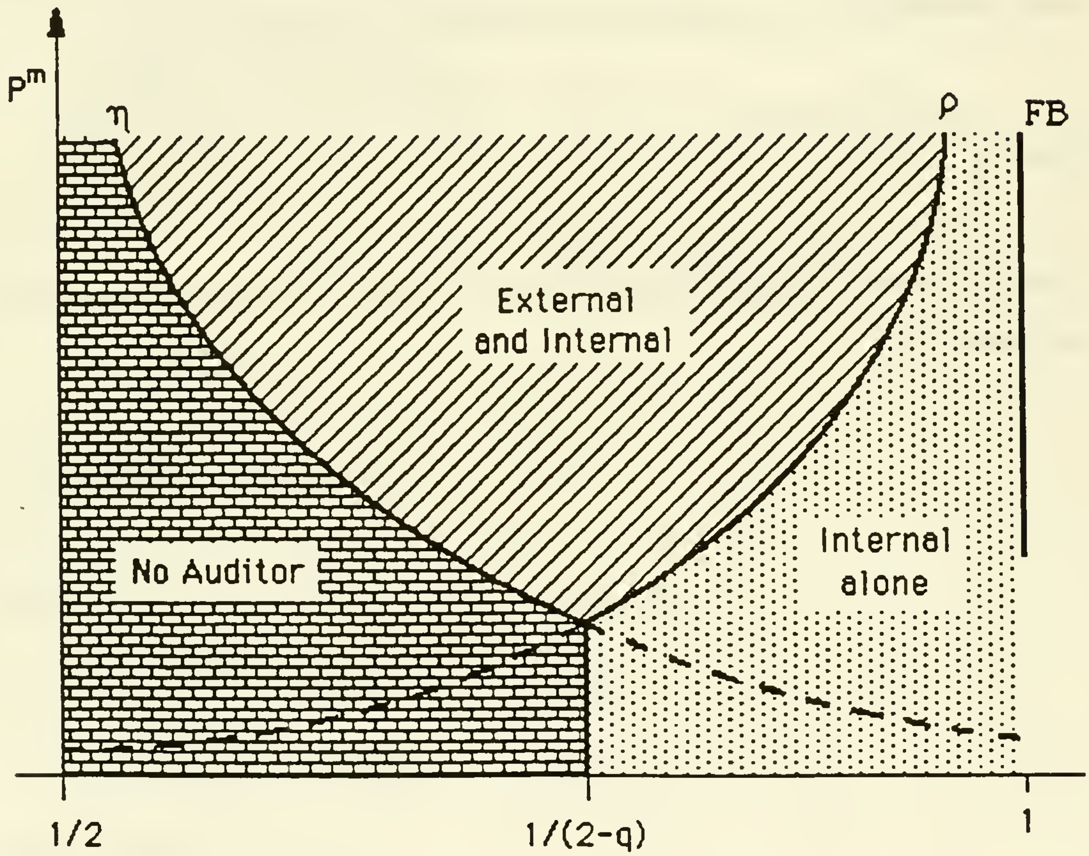

Figure A2. Regions for the use of internal and external auditors

When the principal hires both the internal and the external auditors, $\gamma = 1$, $\delta = \dfrac{p^m}{p^m + p^i}$, $w = 0$ and $P^m > \max\{\eta(r), \rho(r)\}$

We find a situation analogous to the one described in proposition 3. The only difference is that the cost of auditing is now δz instead of z.

Recall that the line $\alpha(r)$ gives the locus of pairs (r,P^m) such that all rent is extracted from the high productivity type of agent and that $\alpha(r)$ is independent of z and P^i. If P^m is between $\eta(r)$ and $\alpha(r)$ the scheme applied (E1) is a straightforward extension of the scheme B1 derived in proposition 3.

Recall that the line $\omega(r)$ gives the locus of pairs (r,P^m) such that the decrease in auditing costs by a reduction in γ is equivalent to the increase in benefits via effort restortion of the low productivity type (see proposition 3). Therefore ω is implicitly defined by

$$P^m = \frac{\Delta\theta\left[2 - \Delta\theta + \sqrt{(2-\Delta\theta)^2 - 16\, z\, \dfrac{P^m}{P^m + P^i}}\right]}{4\,(2r-1)}$$

As in proposition 3, the regions E1 and E2 may not exist. Indeed, line $\alpha(r)$ and $\omega(r)$ could be below $\eta(r)$.

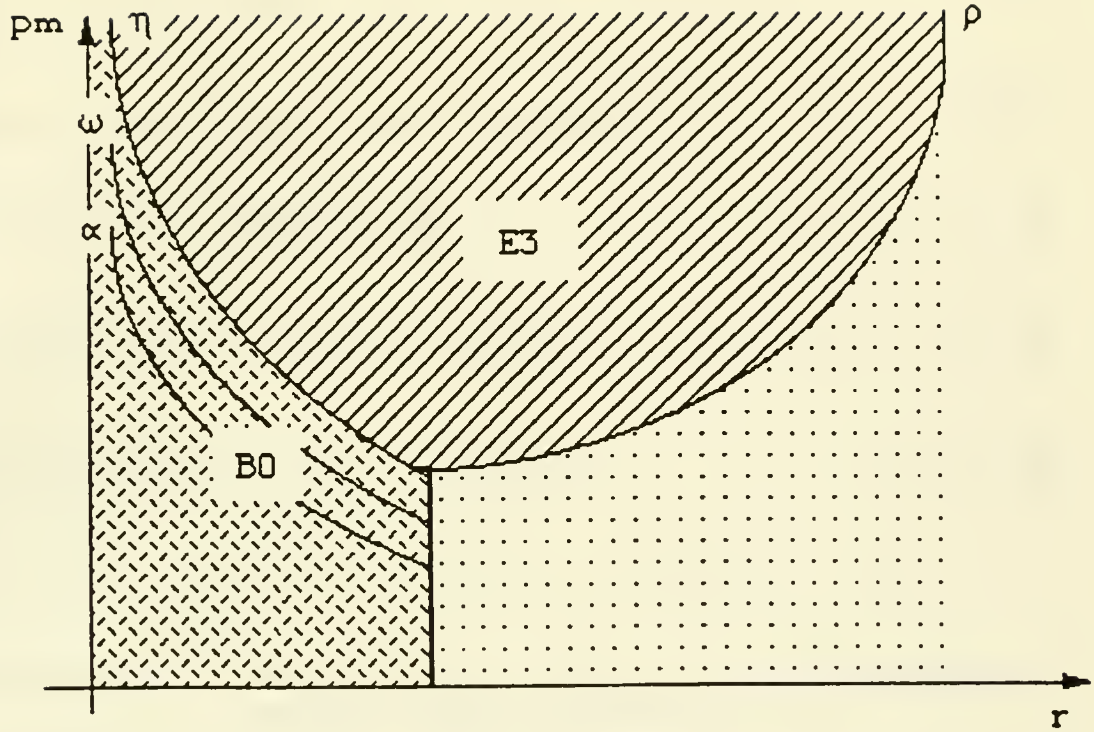

Figure A3. Regions E1 and E2 Do Not Exist

Proof of Lemma 6.1.

The Lagrangian for this problem is:

$$L = q\,\{\theta_1 + e_1 - t_1 + \gamma\,(1\text{-}c)\,(1 - r)\,Pm\} + (1 - q)\,\{\theta_2 + e_2 - t_2\}$$
$$+ \lambda_1\,\{t_1 - \frac{e_1^2}{2} - \gamma\,(1 - r)\,Pm\,(\frac{2 - c}{2})\} \;+ \lambda_2\,\{t_2 - \frac{e_2^2}{2}\}$$
$$+ \lambda_3\,\{t_2 - \frac{e_2^2}{2} - t_1 + \frac{(e_1 - \Delta\theta)^2}{2} + \gamma\,r\,Pm\,(\frac{2 - c}{2})\} + \lambda_4\,\{1 - \gamma\}$$

with the additional constraints:

$e_1 \geq 0 \quad e_2 \geq 0 \quad t_1 \geq 0 \quad t_2 \geq 0$

$\gamma \geq 0 \quad \lambda_1 \geq 0 \quad \lambda_2 \geq 0 \quad \lambda_3 \geq 0 \quad \lambda_4 \geq 0$

The Kuhn-Tucker conditions for maximization are:

$$\frac{\partial \mathcal{L}}{\partial e_1} = q - \lambda_1 e_1 + \lambda_3 (e_1 - \Delta\theta) \le 0 \quad ; \quad e_1 \frac{\partial \mathcal{L}}{\partial e_1} = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = (1 - q) - (\lambda_2 + \lambda_3) e_2 \le 0 \quad ; \quad e_2 \frac{\partial \mathcal{L}}{\partial e_2} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial t_1} = -q + \lambda_1 - \lambda_3 \le 0 \quad ; \quad t_1 \frac{\partial \mathcal{L}}{\partial t_1} = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial t_2} = -(1 - q) + \lambda_2 + \lambda_3 \le 0 \quad ; \quad t_2 \frac{\partial \mathcal{L}}{\partial t_2} = 0 \qquad (4)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = q (1 - c) (1 - r) P^m - \lambda_1 (1 - r) \left(\frac{2 - c}{2}\right) P^m + \lambda_3 r \left(\frac{2 - c}{2}\right) P^m - \lambda_4 \le 0 \quad ; \quad \gamma \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \qquad (5)$$

Plus the constraints and their complementary slackness conditions

If no auditor is used, $\gamma = 0$; therefore $\lambda_4 = 0$ and using (1) to (4) we obtain $\lambda_1 = 1$ and $\lambda_3 = 1 - q$. Plugging those values in (5) yields

$$r \le \frac{2 - c - 2q (1 - c)}{(2 - q) (2 - c) - 2q (1 - c)}$$

Note that when $c = 0$ this condition boils down to $r \le 1/2$ which is never true. In other words, when $c = 0$ it is always optimal to use an auditor. When $c = 1$, the condition becomes $r \le \frac{1}{2-q}$.

Also, when $\gamma = 0$, using (1) to (4) we find that

$$e_1^{B0} = 1 - \frac{1 - q}{q} \Delta\theta$$

i.e., the effort is constant with respect to r and P^m.

When $\gamma = 1$ and MIR2 is not binding, we are in region B1 and using (1) to (4),

$$e_1^{B1} = 1 - \frac{1 - q}{q}\,\Delta\theta$$

and the informational rents are extracted from the high type.

If r or P^m are high enough to bind MIR2,

$$t_1 = \frac{(e_1 - \Delta\theta)^2}{2} + r\,P^m \left(\frac{2 - c}{2}\right)$$

by MIC. Therefore, using MIR1 we can obtain,

$$e_1^{B2} = \frac{\Delta\theta}{2} + P\,\frac{(2r - 1)}{\Delta\theta}\left(\frac{2 - c}{2}\right)$$

Equating e_1^{B1} and e_1^{B2} gives us the line α, separating the two regions B1 and B2:

$$\alpha = \left(\frac{2 - c}{2}\right)\left(\frac{\Delta\theta}{2r - 1}\right)\left[1 - \frac{\Delta\theta}{2q}\,(2 - q)\right]$$

At some point, it may be profitable to decrease γ below 1. If $0 < \gamma < 1$, then $\lambda_4 = 0$ and (5) holds with equality. Also, using (1) to (4) we obtain:

$$e_1^{B3} = 1 - \frac{\Delta\theta\,(1 - r)\,c}{(2r - 1)\,(2 - c)}$$

The region where $0 < \gamma < 1$ is called B3 and the border between B2 and B3 is obtained by solving $e_1^{B2} = e_1^{\ B3}$

$$\beta = \left(\frac{\Delta\theta}{2r - 1}\right)\left(\frac{2}{2 - c}\right)\left[1 - \frac{\Delta\theta}{2} - \frac{\Delta\theta\,(1 - r)}{(2r - 1)\,(2 - c)}\right]$$

Therefore, above β, e_1^{B3} is kept constant below 1 however big the punishment becomes.

Proof of Proposition 6

For this proof we will use the results derived in lemma 6.1 and proposition 4. These two contracts are precisely the possible choices of the principal when facing a free auditor of unknown type: he can allow (lemma 1) or deter (proposition 4) collusion. For each alternative we will derive the profits for the principal and be comparing these profits we will characterize the four main regions.

Derivation of $\underline{\xi}$: The optimal contract of proposition 4 uses the scheme B0 (no auditor) when $r < \frac{1}{2 - q}$. The value of $r = \xi$ of lemma 1. was found to be always smaller than $\frac{1}{2 - q}$. So in the region between ξ and $\frac{1}{2 - q}$ the incentive scheme of lemma 1 dominates the scheme B0. We call the line ξ of lemma 1 $\underline{\xi}$.

Remark: At this point, we have also proved the existence of a region of the paramenter space where it is optimal for the principal to allow collusion. Namely, whenever $r \in (\underline{\xi}, \frac{1}{2 - q})$ regardless of the value of P^m, allowing collusion dominates both other schemes (no auditor and deterring collusion.)

Derivation of $\bar{\xi}_1$: First note that the border between regions B1 and B2 when deterring collusion is below than the border between regions B1 and B2 when allowing collusion; i.e., $\alpha(D) \leq \alpha(A)$ since $\alpha(D) = \frac{2 - c}{2}\,\alpha(A)$ and $c \in (0,1)$.

We can then compare the profits of schemes B1(A) and B1(D) below $\alpha(D)$ and find the border $\bar{\xi}_1$ which equates B1(A) and B1(D). When $r < \bar{\xi}_1$ it will be optimal to allow collusion (using scheme B1(A)) and when $r > \bar{\xi}_1$ it will be optimal to deter collusion (using scheme B1(D)).

When $r > \frac{1}{2 - q}$ and $P^m < \alpha(D)$,

$$\pi(A) - \pi(D) = \frac{P^m}{2}\left\{c\left[q\ (3r - 2) - 2r + 1\right] + 2q\ (1 - r)\right\}$$

which implies that

$$\pi(A) - \pi(D) \geq 0 \quad \text{when} \quad r \leq \frac{2q\ (1 - c) + c}{q\ (2 - 3c) + 2c}$$

and viceversa.

For $r = \overline{\xi}_1$ and $P^m = \alpha(D)$, we saw that $\pi(A) = \pi(D)$. If $\alpha(D) < P^m < \alpha(A)$ and $r = \overline{\xi}_1$ the scheme "allowing" is clearly better since it continues to extract rent at the same constant rate as before while the "deterring" scheme has already extracted all the rent and is restoring effort at a decreasing rate. So the border between "allowing collusion" and "deterring collusion" $(\overline{\xi})$ has to head north-east. Indeed, increasing r makes deterring relatively more profitable because it uses the whole punishment P^m.

Finally, we note that when $r = 1$, $\beta(A) > \beta(D)$. Indeed,

$$\beta(A) = \frac{2}{2 - c}\ \frac{\Delta\theta}{2r - 1}\ \frac{2 - \Delta\theta}{2} > \frac{\Delta\theta\ (4r - \Delta\theta - 2)}{2\ (2r - 1)} = \beta(D)$$

since $\frac{2}{2 - c} > 1$. Also we note that $e_1^{B3(A)} > e_1^{B3(D)}$,

$$e_1^{B3(A)} = 1 - \frac{c}{2 - c}\ \frac{\Delta\theta\ (1 - r)}{2r - 1} > 1 - \frac{\Delta\theta\ (1 - r)}{2r - 1} = e_1^{B3(D)}$$

Therefore, below $\beta(A)$ and above $\beta(D)$ and for $r > \overline{\xi}$ the scheme B3(D) is optimal. And above $\beta(A)$, the scheme B3(A) is optimal.